MAGNETIC RESONANCE IN MEDICINE 3, 778–790 (1986)

Currents and Fields of Thin Conductors in rf Saddle Coils

J. W. CARLSON

Radiologic Imaging Laboratory, University of California, 400 Grandview Drive, South San Francisco, California 94080

Received February 14, 1986

The current distribution on thin conductors and rf field homogeneity for rf coils is described theoretically. After a pedagogical introduction to the techniques and an exact solution for the current or an isolated strip conductor, this article describes current distribution and field uniformity for a variety of conventional and quadrature rf coil designs.

I. INTRODUCTION

At first glance, the description of rf coils for NMR imaging applications appears trivial. Provided that the wavelength of the driving rf is large compared to the length scale of the coil, retardation effects are negligible. In that limit, the problem of computing the electromagnetic field becomes a quasi-static calculation: as is the case for short antennas in the near-field limit (*1*), the current distribution is assumed and the magnetic field is calculated directly by the Biot-Savart law. In this approximation, many of the design parameters of dc coils can be applied directly to rf coils.

This simple description encounters difficulties when faced with a realistic rf coil. Consider first the typical static magnetic field coil. A dc coil typically consists of many fine current paths in series. The current distribution can be assumed uniform across the wire's diameter. Unless one is interested in details of the field very close to a wire (within a few diameters of a wire) the corrections to this approximation are insignificant.

On the other hand, the typical rf coil is constructed quite differently. To reduce self-inductance, an rf coil is usually made with many fewer current loops. And typically, the wires carrying the current are made with some finite extent. The source of the difficulty in calculating the field is that one does not know a priori how the current is distributed across the conductor. For conductors with a width on the order of the diameter of the coil, the effect of altering a presupposed current distribution can have a noticeable effect on the resultant field.

This paper is a theoretical discussion of currents on thin foil conductors and of the resultant fields through a few specific examples. The main results are (1) the derivation of the distribution of current on a thin conductor, (2) the use of this distribution in calculating power loss in the coil, and (3) the calculation of field uniformity. In all situations, I shall assume that the conducting surfaces are infinitely thin and perfectly conducting. The approximation of perfect conductivity is not overly restrictive provided that the skin depth of the conductor is much smaller than its thickness. The assumption

0740-3194/86 $3.00

of infinite thinness of the conductor should be valid provided the thickness is much smaller than the width. The extension of these results to conductors with arbitrary cross sections is a simple extension of this paper.

For this discussion, I shall be interested exclusively in the generic "saddle" coil. By this I mean a coil whose primary current carrying components are parallel to the z axis. In real situations, such coils are constructed by attaching conductors to the surface of a cylinder and parallel to its axis. Such a geometry allows for convenient patient access in whole-body imaging situations. This restriction obviously excludes solenoidal rf coils and many surface coils, as well as the conductors at the ends of saddle coils, for which complications in the complete theoretical discussion of these geometries prevent an analysis in this manner.

II. BASIC PROPERTIES OF THE CURRENT DISTRIBUTION

The problem at hand is the solution of Maxwell's equations subject to boundary conditions. Since the conductors are perfectly conducting, the tangential components of the electric field must vanish. Denote the normal to the conductors by $\hat{n}$. The boundary conditions are stated

$$\mathbf{E} \times \hat{n} = 0.$$

Furthermore, since rf coils are open structures, we also have boundary conditions at infinity. We impose the conditions that the fields vanish at infinity. The vanishing of the tangential components of the electric field on the conductor implies the looser constraint

$$\mathbf{B} \cdot \hat{n} = 0.$$

This condition is just a statement that a good conductor is opaque to rf.

An illustrative example of the consequences of the boundary condition is the single, isolated, flat strip conductor. The cross-sectional geometry is shown in Fig. 1. A surface current flows on the top and bottom of the strip in the positive z direction. Transverse components of the surface current vanish as a result of translational invariance and current conservation.

We can exclude forms of the current distribution if they result in violations of the boundary conditions. For example, assume that the current is uniformly distributed across the conductor. At the center of the conductor, the normal component of the magnetic field vanishes on the surface. However, as one moves away from center, the imbalance in the amount of current on either side results in a net normal component

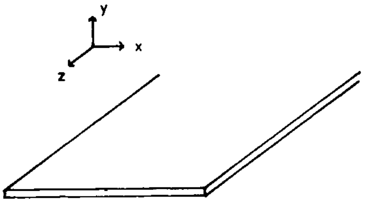

FIG. 1. Geometry of the single, flat strip conductor.

of **B**. Qualitatively, we need more current near the edges of the conductor. As one moves away from center, then the fact that there is more current on one side is balanced by being closer to a concentration of current on the other. On the other hand, the analogue of the "skin effect" is also not a solution. If we assume that all of the current is within one skin depth of either edge, then again we do not cancel the normal components of **B** off center.

This discussion of the previous paragraph illustrates how we could approach the problem. The surface current density must produce no normal component to **B**. The field at x due to a current between x' and dx' is just

$$d\mathbf{B}(x) = \begin{cases} \dfrac{\mu_0}{2\pi}\dfrac{1}{|x-x'|}j(x')dx'\hat{y} & \text{if} \quad x' < x \\ -\dfrac{\mu_0}{2\pi}\dfrac{1}{|x-x'|}j(x')dx'\hat{y} & \text{if} \quad x' > x. \end{cases}$$

We can integrate over the conductor to the left of the point x. (Due to the singularity in the integrand, we must cut off the integration at a distance ε from x.) This integral is from $x' = -b$ to $x' = x - \varepsilon$. The integration to the right of the point x is from $x' = x + \varepsilon$ to $x' = b$. We demand that the net normal component of **B** vanish:

$$\int_{x'=-b}^{x-\varepsilon} \frac{1}{|x-x'|} j(x')dx' - \int_{x'=x+\varepsilon}^{b} \frac{1}{|x-x'|} j(x')dx' = 0. \qquad [1]$$

We could verify explicitly that the different forms of the surface current density discussed earlier are not solutions.

Notice also that the integral equation is independent of frequency. The only assumptions made regarding the frequency were that the skin depth is much less than the thickness of the foil—thus setting a lower limit on the frequency—and that retardation effects are negligible—thus setting an upper limit. If these constraints are satisfied, then current distribution depends only on the geometry of the foil.

Unfortunately, integral Eq. [1] is not readily solvable. The reader may also object to the paradoxical assumptions of having infinitely long conducting foils and neglecting retardation effects. To answer both concerns, the discussion must address the problem of solving Maxwell's equations at a more basic level.

III. SURFACE WAVES

A reasonable starting point for the description of the solution to Maxwell's equations is the assumption that only the local properties of the current distribution determine the fields at any one point near the conductor. In this approximation, the current distribution across a strip should be independent of the location of the ends of the strip provided they are sufficiently far away. We may approximate the physical coil with a set of infinitely long conductors.

The assumed geometry is a series of infinitely long parallel conductors parallel to the z axis. We can generalize the situation and allow for arbitrary cross sections on the conductors.

Without any loss of generality, we can exploit the translation invariance in z and t to do a Fourier decomposition of the components of **E** and **B**.

We may assume the fields are of the form

$$\mathbf{E}(\mathbf{x}, t) = \mathbf{E}(x, y)e^{i(\kappa z-\omega t)}$$
$$\mathbf{B}(\mathbf{x}, t) = \mathbf{B}(x, y)e^{i(\kappa z-\omega t)}.$$

The propagation constant, κ, is not necessarily related to the wave vector $k = \omega/c$. We can express the transverse components of the field in terms of the longitudinal through Maxwell's equations:

$$(\kappa^2 - k^2)\mathbf{E}_t = i\omega[\hat{z} \times \nabla_t B_z] - i\kappa\nabla_t E_z \qquad [2a]$$

$$(\kappa^2 - k^2)\mathbf{B}_t = -ick^2[\hat{z} \times \nabla_t E_z] - i\kappa\nabla_t B_z \qquad [2b]$$

$$\mathbf{E}_t = E_x\hat{x} + E_y\hat{y}, \qquad \mathbf{B}_t = B_x\hat{x} + B_y\hat{y}.$$

All components of $\mathbf{E}$ and $\mathbf{B}$ satisfy a Helmholz equation:

$$[\nabla_t^2 + (k^2 - \kappa^2)]\mathbf{E} = 0$$

$$\nabla_t^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

In addition, E_z satisfies the boundary conditions

$$E_z(\infty) = 0$$

$$E_z(\text{conductor}) = 0.$$

The only solution to the Helmholz equation with these boundary conditions is $E_z \equiv 0$.

In addition, one component of the transverse components of $\mathbf{E}$ vanishes on the conductor. This condition is

$$\hat{n} \times \mathbf{E}_t = 0$$

which implies from Eq. [2a]

$$\hat{n} \times (\hat{z} \times \nabla_t B_z) = 0$$

or

$$\hat{n} \cdot \nabla_t B_z \equiv \frac{\partial}{\partial n} B_z = 0.$$

So we see B_z satisfies a two-dimensional Helmholz equation and the normal derivatives vanish on the boundaries. Again, by uniqueness of solutions, $B_z \equiv 0$ is the only solution. Maxwell's equations for the transverse components reduce to

$$\nabla_t \times \mathbf{E}_t = 0 \qquad \nabla_t \times \mathbf{B}_t = \mu_0 J$$

$$\nabla_t \cdot \mathbf{E}_t = 0 \qquad \nabla_t \cdot \mathbf{B}_t = 0$$

$$\kappa^2 = k^2 \qquad \mathbf{B} = \frac{1}{c}(\hat{z} \times \mathbf{E}_t).$$

The solution to these is

$$\mathbf{E}_t = -\nabla_t \phi$$

where ϕ is a solution of a two-dimensional Laplace equation. On the surface of the conductors, ϕ is a constant. At this point, the treatment of a saddle coil becomes identical to the discussion of the propagation of transverse electric magnetic (TEM) modes on transmission lines (*2*). Notice that this is the complete solution to Maxwell's equations in this specialized geometry. We have had to make no assumptions of neglecting retardation effects whatsoever. Furthermore, these results hold for conductors with arbitrary cross-sectional shape.

For finite conductivity, in general two modes may exist—both a transverse electric (TE) and a transverse magnetic (TM) mode. A discussion of the transition from finite to infinite conductivity for a single circular conductor is best presented by Jones (*3*). The conclusion is that for large conductivity the TE mode decays rapidly and only a single TM mode will propagate. The longitudinal component of **E** is proportional to the resistivity and is much smaller in magnitude than the transverse.

The problem becomes a standard boundary value electrostatics problem. Near the surface of the conductors, **E** is perpendicular to the surface and proportional to the surface charge in the equivalent electrostatics problem. The line integral of **B** along a loop just outside of a conductor is

$$\oint \mathbf{B}\cdot d\mathbf{l} = \oint (\hat{z}\times \mathbf{E}_t)\cdot d\mathbf{l}$$

$\hat{z}\times \mathbf{E}_t$ is parallel to $d\mathbf{l}$ near the surface. Using Ampere's law, we see that the charge on the conductor in the equivalent electrostatics problem is proportional to the current flowing down the conductor in the real problem. It should be emphasized that the surface charge is fictitious. The only source of the fields is the time-varying current density. Furthermore, the line integral of **E** around a closed loop need not vanish unless the loop lies in a constant z plane.

Now we can derive the surface current for the previous example of the isolated flat strip conductor. The electrostatic problem of the infinite flap strip is solved by means of a conformal transformation (*3*). A point in the transverse plane in the original problem has coordinates $w = x + iy$. The function

$$w = \frac{2bv}{1+v^2}$$

maps the w plane to the upper half v plane with the strip forming the real line. In the original problem, field lines extend from the strip to the point at infinity. Since the point at infinity maps to $v = i$, the problem after the mapping is a charged wire at $v = i$ next to an infinite conducting sheet. This is easily solved by images resulting in

$$\phi = (\text{const.})\ln\left(\frac{v-i}{v+i}\right).$$

The electric field of this potential, evaluated at the surface of the conductor, gives us the surface current. This is

$$j(x) = \frac{I}{2\pi}\frac{1}{\sqrt{b^2-x^2}} \qquad -b \leqslant x \leqslant b$$

where I is the total current flowing down the strip. A graph of this distribution is shown in Fig. 2. The current distribution has a mild singularity at the edges. Formally, the singularity is not a problem because it is integrable.

An approximation often used in waveguide theory is to calculate the current density on the walls of the waveguide for the perfect conductors, then use that current density for calculating the power losses in the walls due to finite conductivity. This approximation may be justified on variational grounds as providing an upper bound on the actual power loss in the real waveguide (*2*). Notice that this approximation will not work directly in the strip conductor problem. While the singularity in the current density is integrable, the singularity in the square of the current density is not. We would expect, therefore, that the current density for a real conductor does not have the mathematical singularities at the edges.

In realistic situations the strip conductors are placed on the surface of a cylinder and parallel to its axis. The equivalent electrostatics problem is again exactly solvable for a single isolated conductor. A conformal transformation maps this problem to the previous example. For a curved strip along a circle of radius r in the complex w plane from $w = re^{-i\alpha}$ to $w = re^{i\alpha}$, the mapping

$$v = i\frac{r - w}{r + w}$$

sends the arc to the real line from $-\tan(\alpha/2)$ to $+\tan(\alpha/2)$. The electrostatics problem is reduced to the previous example. The current density of the curved strip conductor is nearly identical to that of the flat strip for subtended angles of less than (roughly) 90°.

IV. SADDLE COILS

The exact solutions of the previous section assume that the presence of the other conductors is not important. We know that in a realistic coil, the current returns on another conductor relatively close to the first. The fields produced by the return path will interact with the currents on the other conductors and alter the distribution.

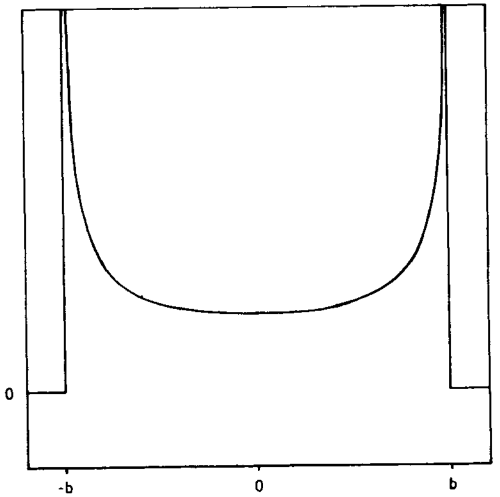

FIG. 2. Current density of flat strip conductor.

Unfortunately, this problem is not solvable exactly and we must resort to numerical methods.

The electrostatic potential with Dirichlet boundary conditions were calculated with the relaxation algorithm (*5*). The results in the remainder of this paper were computed on a 256 × 256 lattice. Since the rf coil is an open structure, we must place the conductors of coil on a circle with a radius of (typically) 60 lattice spacings. The edge of the lattice is grounded. To ensure that the calculated results are not dependent on the presence of the grounded walls of the lattice, the calculations are performed with different placement of the conductors.

An interesting case to consider is the four conductor arrangement of a saddle-coil pair. The geometry and parameters are shown in Fig. 3. The parameters describing the location of the strip are the angles from the symmetry plane to the inside of the strip, θ_i, and to the outside of the strip, θ_o. Also shown in Fig. 3 are the redundant parameters of the center angle, $\theta_c = \frac{1}{2}(\theta_i + \theta_o)$ and the angle subtended by the width of the strip, $\theta_s = (\theta_o - \theta_i)$. The results of the static field analysis of the thin wire coil show that field uniformity occurs when $\theta_c = 30°$ (*7*). The typical rf coil has openings of this size.

Figure 4 shows the numerical solution of the current density for $\theta_s = 30°$ and $\theta_o = 35°$ and 75°. The graphs show the typical behavior: as decreases, the peaks on the inner edges decrease and the current density is more uniform. At $\theta_i = 0$, we would expect to see no inner peak and a single conductor with twice the width. Conversely, as the antiparallel conductors approach one another, the outer peaks increase.

The lattice calculation provides a natural smoothing of the singularities at the edges of the conductors. We can use this property to estimate the power losses of the conductor. I shall use the quantity:

$$q = \mathbf{B}^2(0) \Big/ \int j^2 ds$$

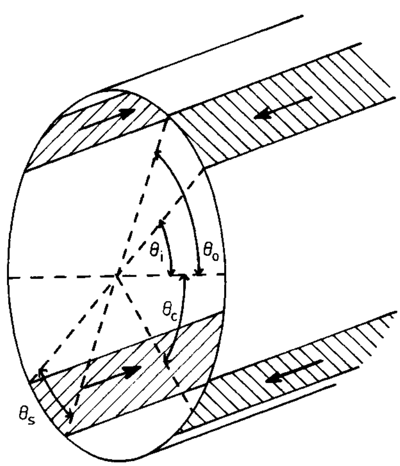

FIG. 3. Geometry of the four strip saddle coil. The conductors extend in *z* direction to plus and minus infinity. Current directions are denoted by arrows.

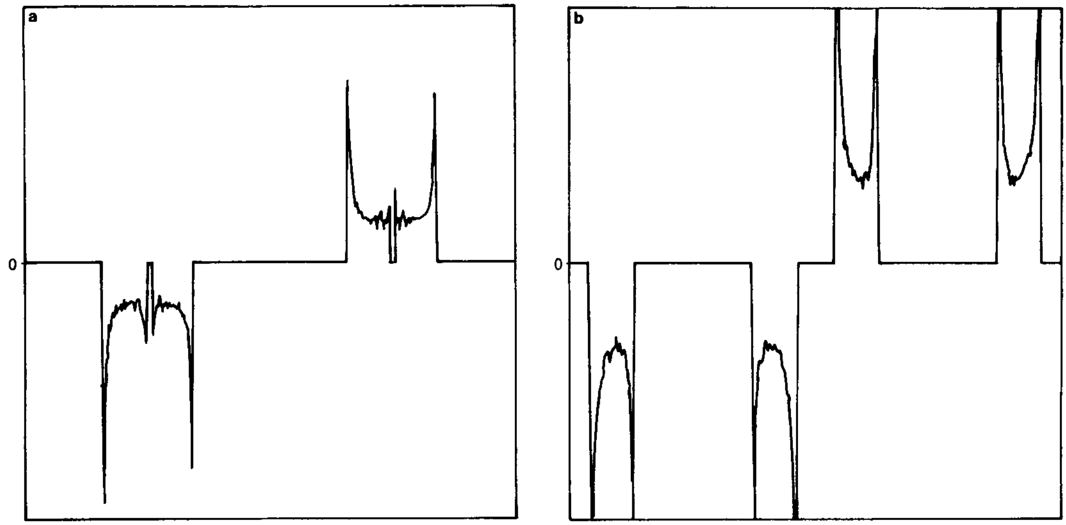

FIG. 4. Numerical solutions for the current density for (a) $\theta_0 = 35°$ and (b) $\theta_0 = 75°$.

where $B(0)$ is the value of the magnetic field at the origin, and j is the computed surface current density. The lattice will introduce artifacts into the evaluation of the integral: since the integrand in the denominator diverges like $1/s$ at the edges of the conductor, the integral diverges like $\log(s)$. We are using the lattice spacing as a cutoff for the integral, so the results of this calculation should contain pieces that diverge (slowly) if the lattice spacing were allowed to decrease. The computed values of q are therefore lattice dependent. The resultant calculations should be taken as a comparative guide in the comparison of different coil geometries.

The results of Fig. 4 suggest that the most uniform current occurs when the gap between the parallel conductors is closed. This is just a slotted cylinder (*6*). Figure 5 shows the results of the calculation of q for different geometries. The units of q are arbitrary; q is a measure of, but not equal to, Q for the ideal, infinitely long rf coil. In particular, Q includes the surface conductivity of the material and therefore is frequency dependent. q is independent of frequency. The results of Fig. 5 are not startling. As expected, power loss is decreased by making wider strips.

The uniformity of the transverse magnetic field is also calculable. The quantity presented in this paper is the relative root mean square deviation in the magnitude in the field over some area:

$$\delta B = \left[\frac{A \int da(B - \bar{B})^2}{\left(\int daB\right)^2} \right]^{1/2} \qquad \bar{B} = \frac{1}{A} \int daB.$$

The area of integration is taken arbitrarily, but not unrealistically, to be a circle with radius of 70% of the radius of the coil. Figure 6 shows the calculations of δB for

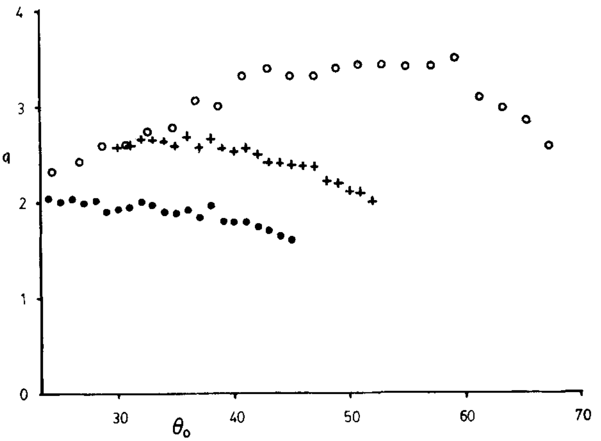

FIG. 5. Calculated values of q for slotted cylinder (circle), saddle coil with $\theta_s = 30°$ (cross), and saddle coil with $\theta_s = 20°$ (dot).

the slotted cylinder, and saddle-coil pairs with θ_s of 20 and 30°. Notice that the location of the center of the strips, θ_c, is close to 30°. As θ_s decreases, the optimum value of θ_c approaches the dc result of $\theta_c = 30°$ (7). For coils with finite extent, the center of the strips moves slightly from this value. More importantly, the uniformity of the field at the optimum foil placement increases by a factor of two as the conductors become wider.

V. SADDLE COILS IN QUADRATURE

An rf coil with two separate saddle coils oriented 90° with respect to one another offer two distinct advantages over the single saddle coil: (1) since circularly polarized

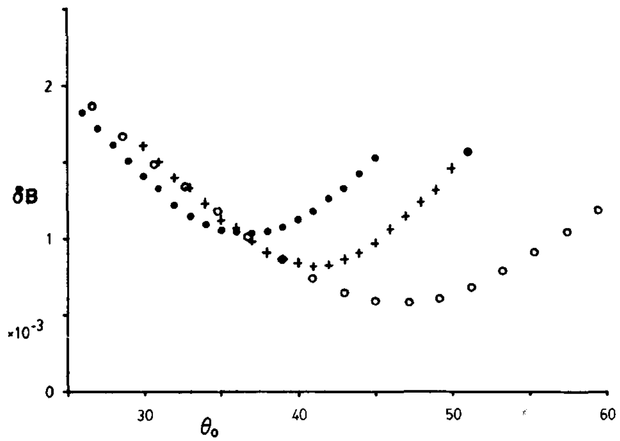

FIG. 6. Field uniformity for slotted tube resonator and two saddle-coil geometries. The symbols are the same as Fig. 5.

rf can be generated, the power necessary to produce a spin flip is reduced, and (2) the detection of a spin echo by a set of coils increases the single to noise ratio by $\sqrt{2}$ (*8*).

The presence of additional conductor alters the currents and fields from the previous arrangement. As before, we can analyze the problem by means of the equivalent electrostatics problem. With the previous arrangement, we knew that equal and opposite currents flowed on each conductor. Since current in the original problem is proportional to charge in the equivalent problem, the electrostatics problem had two conductors at a positive potential and the other two at an equal and opposite value. We may treat the problem by superposition: imagine that a time varying current flows on conductors (A), (B), (C), and (D) of Fig. 7 in the usual sense. Conductors (A′), (B′), (C′), and (D′) are normally 90° out of phase with respect to the other four conductors, however, imagine that these are not being driven at all. By superposition we may calculate the fields with (A), (B), (C), and (D) driven with the primed conductors disconnected and superimpose these with the fields of (A′), (B′), (C′), and (D′) driven out of phase and the unprimed conductors disconnected.

The question is, in the equivalent electrostatics problem what is the potential on the undriven conductors? We can answer that question in this case by symmetry. Conductors (A) and (B) of Figure 7 are at potential ϕ; conductors (C) and (D) are at $-\phi$. By symmetry, the potential on (A′) is the same as (C′) and (B′) the same as (D′). However, the current that flows on (A′) must return on (C′). This implies that the charge on (A′) in the equivalent electrostatics problem must be opposite the charge on (C′). Again, using symmetry arguments, this implies that the potential on (A′) is opposite the potential on (C′). The same holds for (B′) and (D′). Therefore, the potential on the primed conductors vanishes.

Now it is no longer apparent that the slotted cylinder is the ideal rf coil. The presence of the undriven conductor pair is effectively an opaque wall to the rf B field. Consequently, the B field uniformity may be degraded for the quadrature coil. Figure 8

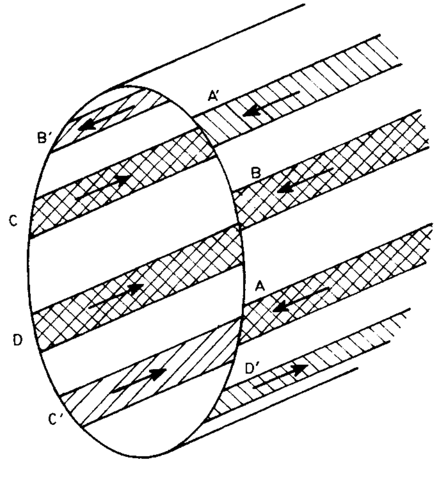

FIG. 7. Geometry of quadrature saddle coil.

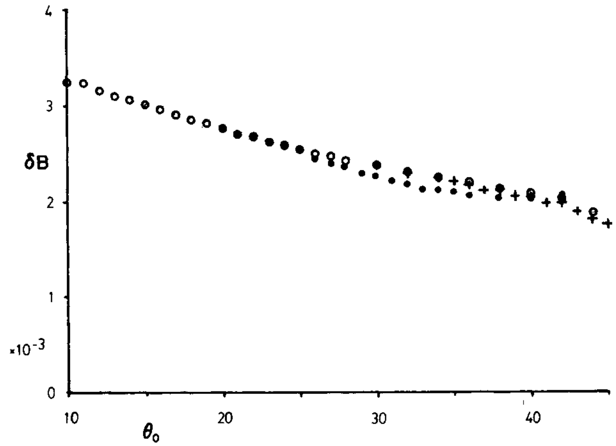

FIG. 8. Uniformity of B field produced by one set of conductors for quadrature coil.

shows the behavior of the field uniformity for this arrangement and confirms this expectation. For greater than 45° the conductors are overlapping. Notice that field uniformity is largely insensitive to the geometry of the coil, but that uniformity is decreased by roughly a factor of 4 over the Fig. 6. In addition, the currents induced on the undriven pair contribute to the power loss in the system. (Even though there is no net current on the undriven pair, eddy currents induced on the conductors dissipate energy. This power loss must be supplied by the driving source.) The calculated q of Fig. 9 shows this effect. In addition, as θ_0 approaches 45°, the primed and unprimed conductors approach one another and q drops sharply.

The curves of Fig. 8 are the results of calculations when current only flows through half of the conductors. This is relevant when the quadrature coil is only driven on

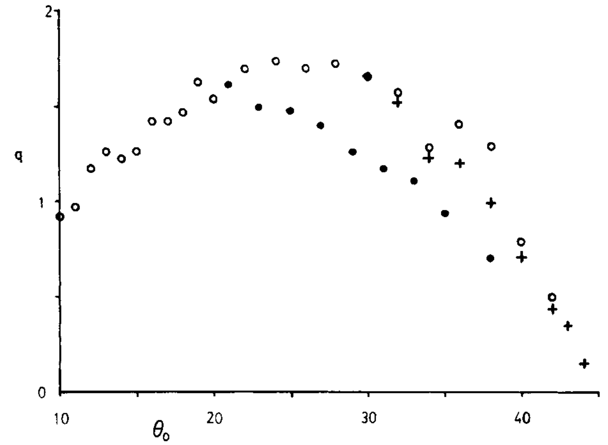

FIG. 9. Calculated values of q for quadrature coil.

one side, which is the case when the QD coil is used as a conventional transmitter. If the coil is used as a quadrature transmitter, or we wish to know the uniformity of sensitivity of the coil, we must properly treat the superposition of the fields (*10*).

By the principle of reciprocity, the induced response of the coil to a precessing spin at a point is proportional to the transverse magnetic field generated by the coil at that point (*9*). The net rotating field is

$$\mathbf{B}_{\text{total}}(r, \theta, t) = \mathbf{B}(r, \theta)e^{-i\omega t} + \mathbf{B}'(r, \theta)e^{-i(\omega t+\pi/2)}$$

where $\mathbf{B}$ is the field generated by the conductors (A), (B), (C), and (D) and $\mathbf{B}'$ is the field generated by the primed conductors. By symmetry, $\mathbf{B}'$ at a point (r, θ) equals in magnitude the field B at $(r, \theta + 90°)$ but the components of $\mathbf{B}'$ rotated by 90° relative to $\mathbf{B}$. The net field in the rotating frame is

$$\mathbf{B}_{\text{total}}(r, \theta) = \mathbf{B}(r, \theta) + \mathbf{B}(r, \theta + 90°).$$

This quantity is the relevant field generated by a true quadrature transmitter. By reciprocity, it is also the quantity which measures the uniformity of response of the coil. Notice that there are two separate effects:

(1) Since the total field is effectively the sum of fields at two locations, the uniformity should increase.

(2) The quantity of interest is a vector sum. If the fields at (r, θ) and $(r, \theta + 90°)$ were equal, the total is just twice the individual fields. The coil is therefore twice as sensitive. If the fields are equal but rotated, the sensitivity of the coil due to spins at that point will not quite double. Therefore the increase in S/N in even the ideal case may not quite be $\sqrt{2}$.

Figure 10 shows the uniformity of the magnetic field for a coil operated in quadrature. As expected the uniformity has improved over all previous designs. However, the shape of the curves is quite different from the previous example. The overall field

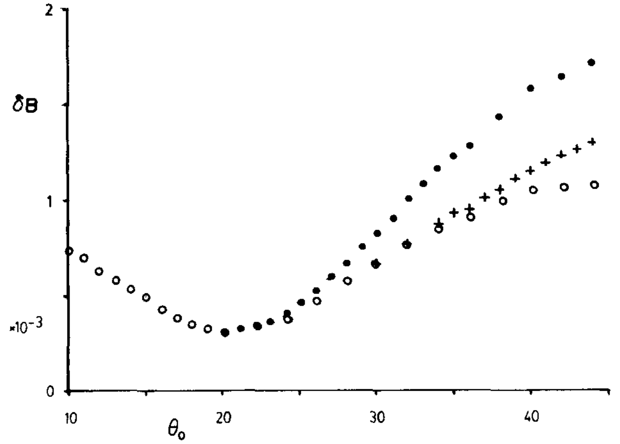

FIG. 10. Uniformity of B field for quadrature coil.

uniformity is greatest in a geometry which had relatively poor uniformity as a conventional rf coil.

The calculation of the mean B field inside the rf coil shows that the average B field in the rotating frame does very nearly double.

VI. CONCLUSIONS

This paper has attempted to demonstrate three main results concerning the current distribution and field uniformity of saddle coils:

(1) The current distribution across a flat strip conductor at high frequency is surprisingly uniform. It is therefore advantageous to make wide current paths.

(2) Field uniformity for a conventional rf coil is maximized using the wide current strips.

(3) The best quadrature rf coil is not simply two conventional rf coils placed on top of one another. It is advantageous to use the properties of the rotating frame magnetic field to design the best quadrature coil. The results of these calculations show that the best geometry is the slotted cylinder with $\theta_0 \approx 25°$. The q for this geometry is roughly half the q for a conventional receiver coil. If the thermal noise in the rf coil were the dominant source of noise in an NMR experiment, then the degradation in q would result in an increase in noise. In such a situation S/N would not increase by the ideal amount.

ACKNOWLEDGMENT

This work is supported in part by Diasonics, (MRI) Inc.

REFERENCES

1. J. D. JACKSON, "Classical Electrodynamics," Wiley, New York, 1975.
2. R. E. COLLIN, "Field Theory of Guided Waves," McGraw-Hill, New York, 1960.
3. D. S. JONES, "The Theory of Electromagnetism," Macmillan, New York, 1964.
4. E. WEBER, "Electromagentic Fields: Theory and Applications," Vol. 1, Wiley, New York, 1950.
5. W. E. MILNE, "Numerical Solution of Differential Equations," Dover, New York, 1970.
6. H. J. SCHNEIDER AND P. DULLENKOPF, *Rev. Sci. Instrum.* **48,** 68 (1977); A. LEROY-WILLIG *et al., Magn. Reson. Med.* **2,** 20 (1985).
7. D. M. GINSBERG AND M. J. MELCHNER, *Rev. Sci. Instrum.* **41,** 122 (1970).
8. C.-N. CHEN, D. I. HOULT, AND V. J. SANK, *J. Magn. Reson.* **54,** 324 (1983).
9. D. I. HOULT AND R. E. RICHARDS, *J. Magn. Reson.* **24,** 71 (1976).
10. D. I. HOULT, C.-N. CHEN, AND V. J. SANK, *Magn. Reson. Med.* **1,** 339 (1984).