Analysis and design of compact, static Fourier-transform spectrometers

Carole C. Montarou and Thomas K. Gaylord

A new, to our knowledge, analytical method is presented to characterize the performance of modified-Wollaston-prism-based compact, static Fourier-transform spectrometers. With the aid of an exact raytracing method for birefringent media, the interference of the two wave fronts produced by the beam splitter is computed at an arbitrarily positioned detector array. It is shown that a compact, static Fourier-transform spectrometer employing a single modified Wollaston prism can be designed such that the fringes are perpendicular to the incident beam. The effects of aperture size, coherence of the source, and incidence angle on the resulting interferogram are quantified. © 2000 Optical Society of America *OCIS codes:* 070.2590, 120.3180, 300.6190, 120.4570.

1. Introduction

Fourier-transform spectrometers (FTS's) are used extensively as diagnostic instruments. They allow for the accurate measurement of the absorption spectra of gases, liquids, and solids.¹ FTS's can be used as laser wavemeters to determine accurately the wavelength of tunable lasers.² The Michelson interferometer format of the FTS is often used for laser wavemeter applications. In these cases, a temporal interferogram is formed when one of the mirrors is scanned. An inverse Fourier transform of this interferogram allows for the retrieval of the power spectrum.³ However, the scanning mechanism requires high precision, and the moving parts are a potential source of noise in the detected signal. This leads to expensive designs and substantial bulk. These factors have provided motivation for the design of compact, static FTS instruments with no moving parts. In a compact, static FTS, a spatial interferogram is formed and recorded with a detector array. The most recent designs of compact, static FTS's are based on Wollaston prisms.^{1,2,4–7}

Conventional Wollaston prisms (CWP's) and modified Wollaston prisms (MWP's) are composed of two triangular pieces of uniaxial birefringent crystal whose optic axes are perpendicular to each other as shown in Fig. 1. This produces the splitting of an incident linearly polarized beam into two orthogonally polarized beams. Placed between crossed polarizers, they produce interference fringes at the plane of apparent splitting (PAS). This is inside the prism in the case of the CWP [Fig. 1(a)].⁸ A MWP, on the other hand, forms the interference fringes outside the prism because the optic axis of one of the triangular wedges is inclined relative to the external faces of the prism as shown in Fig. 1(b). An instantaneous recording of the interferogram is therefore possible when a detector array is placed in coincidence with the interference fringes of the MWP.^{2,4,7} This leads to inexpensive, compact FTS designs with no moving parts.^{1,2,4-7}

Padgett and Harvey⁵ designed a FTS based on a single CWP whose interference fringes are projected onto a detector array by use of a lens. The FTS can also be designed with two CWP's or one MWP. In both of these cases, the lens used to project the fringes on the CCD is eliminated because the interference fringes are formed outside of the prism.^{1,2,4,5,7,8} More recently, Prunet *et al.*⁹ developed an analytical method to calculate the total optical path difference as a function of the angle of incidence in a two-CWP-based FTS. These Wollaston-prism-based FTS's are used in a wide range of applications including a fiber-coupled laser wavemeter designed by Steers *et al.*² and an ultraviolet spectrometer for gas detection designed by Courtial *et al.*¹

However, an accurate analysis method based on the operational parameters of the compact, static FTS is not available. There is a need to be able to

The authors are with the School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, Georgia 30332-0250.

Received 22 February 2000; revised manuscript received 30 June 2000.

^{0003-6935/00/315762-06\$15.00/0}

^{© 2000} Optical Society of America



Fig. 1. (a) CWP made from a positive uniaxial birefringent material. Two triangular pieces of birefringent material are cemented together with their optic axes (o.a.'s) orthogonal to each other. In one piece, the optic axis is perpendicular to the plane of incidence, whereas in the other piece the optic axis lies in the plane of incidence and is parallel to the entrance and exit faces of the prism. A linearly polarized beam is incident on the entrance face and is split into two orthogonally polarized beams as shown. The wedge angle between the normal to the exterior interfaces and the middle interface is γ . (b) MWP made from positive uniaxial birefringent material. The two optic axes are again perpendicular to each other. However, the optic axis that lies in the plane of incidence is inclined with respect to the entrance and exit faces of the prism. The inclination angle between the normal to the exterior interfaces and the latter optical axis is δ .

characterize directly the resulting interferogram as opposed to the indirect characterization in terms of optical path differences.^{1,2,4–7,9} A direct simulation of the interferogram would allow the evaluation of the effects of the aperture size, angle of incidence, coherence of the source, and other operational parameters.

In this paper an exact ray-tracing method⁸ is adapted to predict the interferogram of an arbitrary static FTS. Furthermore, it is shown that an improved FTS can be designed based on one MWP inclined so that the resulting interference fringe plane is perpendicular to the axis of the optical system (AOS). The effects of overfilling the aperture, the coherence of the source, the material dispersion, and the angle of incidence are quantified.

2. Analysis

A. Fourier-Transform Spectrometer Design

The most recent designs of FTS's are based on CWP's and MWP's. A CWP is composed of two triangular wedges of uniaxial birefringent material whose optic axes are perpendicular to each other. These prisms split an incident linearly polarized beam into two orthogonally polarized transmitted beams.⁸ With a CWP, this splitting occurs inside the prism and produces interference fringes when the prism is placed between crossed polarizers [Fig. 1(a)]. With a MWP, this splitting occurs outside the prism because the optic axis of one of the wedges is inclined relative to the external faces of the prism⁸ [Fig. 1(b)]. The interference fringe plane can also be located outside when two CWP's are used.^{1,2,4,5,9} When a detector array is placed in coincidence with the fringes of the MWP or of the two CWP's, an instantaneous recording of the interferogram is possible; and when an inverse Fourier transform is performed, the power spectrum can be retrieved. This is the basis of the design of compact, static FTS's.

The linear variation of the optical path difference along the fringe plane produces the interferogram. This optical path difference is a function of the splitting angle $\Delta\beta$ between the two orthogonally polarized beams exiting the prism. Therefore, for a source of free-space wavelength λ , the fringe spacing Λ of the interferogram is given by

$$\Lambda = \frac{\lambda}{2\sin(\Delta\beta/2)}.$$
 (1)

In Subsection 2.B., for the first time to our knowledge, a new analytical method is described that allows for the evaluation of compact, static FTS's by fully characterizing the resulting interferogram. The effects such as aperture size, beam waist, coherence of the source, and angle of incidence can thus be quantified. Furthermore, we show that it is possible to design an improved compact, static FTS whose fringe plane is perpendicular to the AOS.⁸

B. Interferogram Characterization

To characterize fully the interferogram formation, ray tracing of the two orthogonally polarized beams inside the MWP must be performed. A detailed analysis of the ray tracing can be found in Ref. 8. Input light is incident upon the MWP as shown in Fig. 2. For the analysis, the incident beam is divided into a number of individual rays.

Uniaxial media split a single incident linearly polarized ray into two orthogonally polarized rays: an ordinary ray whose electric field is perpendicular to the optical axis of the birefringent medium and an extraordinary ray whose magnetic field is perpendicular to the optical axis of the birefringent medium. The ray whose electric field is perpendicular to the plane of incidence is the transverse-electric (TE) ray and the ray whose magnetic field is perpendicular to the plane of incidence is the transverse-magnetic (TM) ray. The TE ray (dot) and the TM ray (double arrow) are shown in Fig. 2 after exiting the MWP. An important parameter of the MWP is the output splitting angle between the two orthogonally polarized beams. This is computed by an exact raytracing analysis⁸ in each wedge of the MWP. The phase-matching condition is applied at the interface between regions: when the allowed wave-vector surface cross section is used, the component of the wave vector along the interface must be equal in each region.⁸ The magnitude and direction of the wave vector for each polarization is calculated. When the allowed wave-vector surface cross section is two circles, e.g., as in Fig. 2 in the first region of the MWP, the wave vector and the Poynting vector have the same direction. However, in the second region of the MWP, the allowed wave-vector surface cross section



Fig. 2. Operation of a FTS based on a MWP. The MWP is tilted so that the light is incident at an angle α . The input light is divided into TE- and TM-polarized beams after passing through the prism. The TE- (dot) and TM- (double arrow) polarized beams resulting from two incident beams are shown. The prism material is a negative birefringent material such as calcite. Placed between crossed polarizers, interference fringes are formed at the PAS (dashed line). An x, y system is associated with the MWP. The optic axis (o.a.) of the first wedge of the prism is perpendicular to the plane of incidence whereas it lies in the plane of incidence in the second wedge of the prism. The AOS is shown. The plane of the detector array is assigned with a coordinate system u_{det}, v_{det} . Point B is the intersection point between the PAS and the AOS. The angles between the TE-polarized beam and the AOS and the TM-polarized beam and the AOS are, respectively, ξ_4^{TE} and ξ_4^{TM} .

of the extraordinary ray (TM) is an ellipse, and the direction of the Poynting vector is perpendicular to the allowed wave-vector surface.⁸ An x, y system of axes is assigned to the MWP as shown in Fig. 2. Knowing the direction angles for each wave vector in each region, the equations of the rays are calculated in the x, y system.⁸ The ray equations of the TE and TM rays shown in Fig. 2 are

$$y_4^{\text{TE}} = (x_4^{\text{TE}} - w) \tan \beta_4^{\text{TE}} + y_{3,4}^{\text{TE}},$$

$$y_4^{\text{TM}} = (x_4^{\text{TM}} - w) \tan \beta_4^{\text{TM}} + y_{3,4}^{\text{TM}},$$
 (2)

where *w* is the total thickness of the MWP, β_4^{TE} is the angle between the TE ray and the *x* axis of the *x*, *y* system; β_4^{TM} is the angle between the TM ray and the *x* axis of the *x*, *y* system; $y_{3,4}^{\text{TE}}$ and $y_{3,4}^{\text{TM}}$ are, respectively, the *y* coordinates of the points of intersection of the TE and TM rays with the last interface of the MWP.

The point of intersection of the TE and TM beams, respectively, with the detector array in the x, y system are x_{det}^{TE} , y_{det}^{TE} and x_{det}^{TM} , y_{det}^{TM} . The equation of the detector array shown in Fig. 2 is given by

$$y_{\rm det} = m_{\rm det} x_{\rm det} + b_{\rm det},\tag{3}$$

where m_{det} and b_{det} are calculated for a given FTS configuration. Equations (2) allow for the computa-

tion of the points of intersection of, respectively, TE and TM rays with the detector array

$$x_{det}^{TE} = \frac{b_{det} - w \tan \beta_4^{TE} + y_{3,4}^{TE}}{m_{det} - \tan \beta_4^{TE}},$$

$$x_{det}^{TM} = \frac{b_{det} - w \tan \beta_4^{TM} + y_{3,4}^{TM}}{m_{det} - \tan \beta_4^{TM}},$$

$$y_{det}^{TE} = (x_{det}^{TE} - w) \tan \beta_4^{TE} + y_{3,4}^{TE},$$

$$y_{det}^{TM} = (x_{det}^{TM} - w) \tan \beta_4^{TM} + y_{3,4}^{TM}.$$
(4)

We calculated the coordinates along the plane of the detector array, v_{det}^{TE} and v_{det}^{TM} , for TE and TM, respectively, by computing the distances between point B and the points of intersection with the detector array of, respectively, TE and TM polarized beams. The normalized electric fields for TE and TM polarized beams at the plane of the detector array are given by

$$E_{\rm TE} = \exp[j(\omega t - k_{v_{\rm det}}^{\rm TE} v_{\rm det}^{\rm TE})],$$

$$E_{\rm TM} = \exp[j(\omega t - k_{v_{\rm det}}^{\rm TM} v_{\rm det}^{\rm TM})],$$
(5)

where ω is the radian frequency, t is the time, and $k_{v_{\rm det}}^{\rm TE}$ and $k_{v_{\rm det}}^{\rm TM}$ are given by

$$k_{v_{\text{det}}}^{\text{TE}} = k_0 \sin \xi_4^{\text{TE}}, \quad k_{v_{\text{det}}}^{\text{TM}} = k_0 \sin \xi_4^{\text{TM}},$$
 (6)

where k_0 is the wave-vector magnitude in free space, $\xi_4^{\text{TE}} = \beta_4^{\text{TE}} - \alpha$ and $\xi_4^{\text{TM}} = \beta_4^{\text{TM}} - \alpha$, with α the angle of incidence upon the MWP (Fig. 2). The above procedure is performed for each of the rays comprising the input beam. The incident input light is taken to have a Gaussian distribution in space of its electric field. Therefore the intensities of the split TE- and TM-polarized beams are weighted correspondingly. The total electric field along the detector array is given by the sum of all the electric fields calculated as described above. We obtained the intensity by multiplying the total electric field by its complex conjugate. This therefore leads to the computation of the interferogram observed in the plane of the detector array.

3. Plane of Apparent Splitting Perpendicular to the Axis of the Optical System

The above procedure is applied to design a compact, static FTS with the PAS of the MWP perpendicular to the AOS and the detector array placed along the PAS. The MWP is rotated correspondingly, i.e., the angle of incidence α (Fig. 2) upon the MWP is calculated with the equations derived in Ref. 8 so that the PAS is perpendicular to the AOS. The prism is chosen to be made of calcite, with $\gamma = 88.4^{\circ}$ and $\delta = 80^{\circ}$. With the procedure developed in Ref. 8, the angle of incidence α_{\perp} necessary to make the PAS perpendicular to the AOS is calculated to be $\alpha_{\perp} = 4.60792^{\circ}$.

A. Aperture Effect

The method presented in Section 2 is applied to a compact, static FTS with the design parameters defined above. The wavelength of the source is equal



Fig. 3. Interferogram produced by a source of wavelength $\lambda = 500$ nm. The MWP is made of calcite and is tilted so that its PAS is perpendicular to the AOS. The wedge angle between the *x* axis of the prism and its middle interface is $\gamma = 88.4$ deg, and the optic axis inclination angle between the *x* axis of the prism and the optic axis in the second wedge of the prism is $\delta = 80$ deg. The incident beam waist is $\omega_0 = 5$ mm.

to 500 nm. The interferogram is calculated for three different values of beam waist: $\omega_0 = 5$, 10, and 15 mm. In these three cases, the beam is incident on the MWP shown in Fig. 2 from y = -5 mm to y = 5 mm. The three resulting interferograms are shown in Figs. 3, 4, and 5. In each case, the number of fringes in the interferograms is in good agreement with Eq. (1). At 500 nm, the number of fringes per millimeter is 19.5 mm⁻¹ according to Eq. (1). The interferograms in Figs. 3, 4, and 5 produce nearly this number of fringes per millimeter. The ideal interferogram for one wavelength is expected to be a constant-amplitude sinusoid. However, Fig. 3 shows that this is not the case with a 10-mm entrance



Fig. 4. Interferogram produced by a source of wavelength λ equal to 500 nm. The MWP is made of calcite and is tilted so that its PAS is perpendicular to the AOS. The wedge angle between the *x* axis of the prism and its middle interface is $\gamma = 88.4$ deg, and the optic axis inclination angle between the *x* axis of the prism and the optic axis in the second wedge of the prism is $\delta = 80$ deg. The incident beam waist is $\omega_0 = 10$ mm.



Fig. 5. Interferogram produced by a source of wavelength λ equal to 500 nm. The MWP is made of calcite and is tilted so that its PAS is perpendicular to the AOS. The wedge angle between the *x* axis of the prism and its middle interface is $\gamma = 88.4$ deg, and the optic axis inclination angle between the *x* axis of the prism and the optic axis in the second wedge of the prism is $\delta = 80$ deg. The incident beam waist is $\omega_0 = 15$ mm.

aperture and a 5-mm input beam waist. The resulting interferogram has the appearance of an interferogram that has a source containing two wavelengths. Figure 5 shows that this effect is reduced when the aperture is overfilled with an input beam waist of 15 mm.

Furthermore, it is known that the interferogram produced by two spectral lines is composed of two periods, i.e., fine fringes under a larger period envelope. Therefore the resolution depends on the length of the detector array. To resolve two wavelengths, the detector array must be long enough to record the total length of the larger envelope. This also depends on the width of the input beam. The width of the envelope of the interferogram produced by two spectral lines is a function of the phase-shift difference between the beams across the plane of the detector. The resolution of the compact, static FTS could therefore be improved by optimizing the phaseshift difference that, in turn, depends on the MWP design parameters.

B. Coherence Effect

The same method is now applied with a source that has a Gaussian spectral distribution from 495 to 505 nm and centered at 500 nm. The waist of this Gaussian spectral distribution is chosen to be 3 nm. The interferogram produced by such a source is calculated for a compact, static FTS with the same design parameters as the ones above and with a beam waist ω_0 equal to 15 mm. The aperture length is 10 mm, i.e., the coordinate y of the incident beam goes from -5 to 5 mm. The interferogram for each wavelength present in the source is calculated and weighted with the corresponding coefficient. We obtained the total interferogram by adding the individual interferograms for each wavelength. The result



Fig. 6. Interferogram produced by a source with a Gaussian spectral distribution centered at 500 nm and a waist of 3 nm. The MWP is made of calcite and is tilted so that its PAS is perpendicular to the AOS. The wedge angle between the *x* axis of the prism and its middle interface is $\gamma = 88.4$ deg, and the optic axis inclination angle between the *x* axis of the prism and the optic axis in the second wedge of the prism is $\delta = 80$ deg. The incident beam waist is $\omega_0 = 15$ mm. The lower dashed curves show the interferogram when the material dispersion is not taken into account. The upper dashed curves show the interferogram when a material exhibiting anomalous dispersion is used.

of this procedure is shown in Fig. 6. When the coherence of the source is incorporated, the interferogram is not a constant-amplitude sinusoid as would be expected from a source containing only one spectral line. The envelope has a shape of a Gaussian function. Figure 6 also shows the shape of the envelope when the material dispersion is not taken into account. A comparison of the two shows that material normal dispersion somewhat degrades the performance of compact, static FTS's. A hypothetical interferogram produced by a material exhibiting an equivalent anomalous dispersion is also shown in Fig. 6. The envelope is, in this case, broader than those of the two previous cases. Therefore use of material exhibiting anomalous dispersion would improve the performance of static FTS and tend to compensate for the coherence effect because it is closer to the ideal case of a constant-amplitude sinusoid.

C. Angle of Incidence Effect

The effect of the angle of incidence can be quantified by use of the present method. The same prism design parameters as the ones in Subsections 3.A and 3.B are used. The input beam waist ω_0 is equal to 15 mm. The source spectral distribution is identical to the one used in Subsection 3.B. The method was applied for various angles of incidence. The interferograms obtained for an angle of incidence $\alpha = +5^{\circ}$ and -5° are virtually identical to the interferogram shown in Fig. 6. This illustrates that the performance of the FTS design presented is largely insensitive to variations in the angle of incidence. The only change observed is that the location of the intensity maximum of the interferogram is shifted

along the detector array. The positions of this maximum along the detector array for $\alpha = -5^{\circ}$, 0°, and $+5^{\circ}$ are, respectively, $v_{det} = -0.1169$, -0.1027, and 0.0304 mm. At normal incidence, a beam incident at $y_0 = 0$ mm upon the entrance face of the prism is deviated down relative to the axis of the optical system after passing through the prism. Therefore the intensity maximum of the input beam is shifted down at the output of the prism, and its position on the detector array is slightly below the position of point B shown in Fig. 2. Consequently, v_{det} is slightly negative. For $\alpha = +5^{\circ}$, the input beam is deviated up relative to the axis of the optical system, and for $\alpha =$ -5° the input beam is deviated down relative to the axis of the optical system. Other than this slight shift of the intensity maximum of the interferogram, the angle of incidence largely does not affect the interferogram produced by a Gaussian source.

4. Conclusion

A new modeling approach has been presented for direct characterization of the interferogram produced by a compact, static FTS based on a MWP. Applying exact ray tracing through the MWP, we calculated the normalized electric fields of the TE- and TMpolarized beams in the plane of the detector array. This leads to an accurate calculation of the intensity distribution along the detector array. It has been shown that a compact, static FTS can be designed by use of a single MWP and configured such that its fringe plane is perpendicular to the incident beam.

The effects of the aperture size, coherence of the source, and angle of incidence have been quantified. The interferogram is more accurate when the aperture is overfilled as was shown in Figs. 3–5. A smaller beam waist produces an interferogram with less wavelength resolution. Furthermore, the resolution depends on the width of the detector and the width of the input beam incident on the prism. The interferogram of a source with a Gaussian spectral distribution has been calculated and compared to the ideal case of a single spectral line. It has been shown that material normal dispersion somewhat degrades the performance of a compact, static FTS. The effect of the angle of incidence has been quantified in terms of the measured interferogram. It was shown that the interferogram was insensitive to the angle of incidence variations. It only shifts the interferogram slightly relative to the interferogram obtained at normal incidence. The improved compact, static FTS design presented may be useful in a wide variety of present and future applications.

References

- J. Courtial, B. A. Patterson, W. Hirst, A. R. Harvey, A. J. Duncan, W. Sibbett, and M. J. Padgett, "Static Fouriertransform ultraviolet spectrometer for gas detection," Appl. Opt. 36, 2813–2817 (1997).
- D. Steers, W. Sibbett, and M. J. Padgett, "Dual-purpose, compact spectrometer and fiber-coupled laser wavemeter based on a Wollaston prism," Appl. Opt. **37**, 5777–5781 (1998).

- R. J. Bell, Introductory Fourier Transform Spectroscopy (Academic, London, 1972).
- J. Courtial, B. A. Patterson, A. R. Harvey, W. Sibbett, and M. J. Padgett, "Design of a static Fourier-transform spectrometer with increased field of view," Appl. Opt. 35, 6698-6702 (1996).
- M. J. Padgett and A. R. Harvey, "A static Fourier-transform spectrometer based on Wollaston prisms," Rev. Sci. Instrum. 66, 2807–2811 (1995).
- M. J. Padgett, A. R. Harvey, A. J. Duncan, and W. Sibbett, "Single-pulse, Fourier-transform spectrometer having no moving parts," Appl. Opt. 33, 6035–6040 (1994).
- B. A. Patterson, M. Antoni, J. Courtial, A. J. Duncan, W. Sibbett, and M. J. Padgett, "An ultra-compact static Fouriertransform spectrometer based on a single birefringent component," Opt. Commun. 130, 1–6 (1996).
- C. C. Montarou and T. K. Gaylord, "Analysis and design of modified Wollaston prisms," Appl. Opt. 38, 6604–6616 (1999).
- 9. S. Prunet, B. Journet, and G. Fortunato, "Exact calculation of the optical path difference and description of a new birefringent interferometer," Appl. Opt. **38**, 983–990 (1999).