Calibration of a computer-controlled precision wavemeter for use with pulsed lasers

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The design of a pulsed wavemeter to monitor the high-precision tuning of pulsed (as well as cw) laser sources is presented. This device is developed from a combination of silver-coated Fabry–Perot etalons with various plate spacings. These etalons provide stepwise refinement of the wavelength to be measured. The wavemeter is controlled by a computer through a CAMAC interface, which measures the absolute wavelength in the visible with an accuracy of 2 parts in 10^8 . The time required for data acquisition and computation to measure the refined wavelength with a single 2-MHz CPU is less than 100 ms. We describe the calibration of the instrument over the wavelength range 400–850 nm. We obtain the required calibration lines by locking lasers on hyperfine transitions of iodine, uranium, rubidium, and cesium. Methods to reduce the number of calibration lines required for calibration of the system are described. The expected wavelength-dependent phase shift of the silver coatings is compared with that measured for the etalon following calibration. The differences are larger than expected because of either optical aberations or the use of centroids to measure the fringe position. © 1999 Optical Society of America

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1. Introduction

Tunable laser systems such as dye, diode, and Ti: sapphire, which span the visible and extend to the UV and IR regions, are common in advanced laser laboratories. Precision tuning of such cw lasers is monitored by wavemeters based on measuring the fringe difference relative to a reference laser with a scanning interferometer.

If we define accuracy as the ratio of the standard error in measuring the wavelength to the wavelength, the best wavemeter for cw lasers was built^{1,2} by Schawlow and colleagues with an accuracy of a few parts in 10⁹. However, most nonlinear optics^{3,4} experiments use pulsed lasers. In high-resolution laser spectroscopy experiments the wavelength of the atomic or molecular transition under study is derived from measurements of the wavelength of one or more laser lines involved in the experiment. Unfortunately, the tuning of pulsed lasers cannot be monitored by a scanning interferometer but rather uses Fabry–Perot etalons. Such a wavemeter can monitor both cw and pulsed lasers. Further, single-mode operation of the laser is not required by Fabry–Perot etalons.

Byer et al.⁵ designed a pulsed wavemeter; Fischer et al. presented⁶ an improved version of Byer's design. Konishi *et al.* presented a similar design.⁷ All these attempts achieved precisions of 1 part in 107. Somewhat later, Snyder⁸ and Snyder et al.⁹ applied the Fizeau interferometer to the measurement of the wavelengths of pulsed lasers. The oscillating fringes along the wedge of a Fizeau interferometer allow one to measure an approximate wavelength. This wavelength can then be used to determine the absolute phase difference at a reference point along the wedge where the thickness has been calibrated. This last measurement of wavelength is similar to measurement with a Fabry-Perot etalon and is subject to similar errors. The calibration of Fizeau wavemeters are sensitive to wave-front curvature and the maintenance of a precise angle of incidence to the wedge. The design of Garner¹⁰ minimizes the effect of wavefront curvature. Gray *et al.*¹¹ found additional reduction in this sensitivity by a small rotation of the detector axis relative to the axis of the wedge. They determined a resolution of 2–3 parts in 10⁷ with an absolute accuracy of 1 part in 10^6 . Recently others have presented compact designs with somewhat inferior precision.¹²⁻¹⁴ A wavemeter with improved absolute accuracy could

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be built by a combination of a Fizeau interferometer with one or more Fabry–Perot etalons with increasing thickness, or one could choose to use all etalons as in Byer's original design. Etalons have the advantage that their calibration is insensitive to the wave-front curvature and the pointing direction of the incident laser. A wavemeter employing both etalons and a Fizeau interferometer would also require two different types of data-reduction technique.

We present an improved version of the designs of Byer and Fischer et al. To reduce the temperature and wavelength dependence of the refractive index between etalon plates we use evacuated, silvercoated etalons. Silver coatings were chosen because of their high reflectivity over a broad spectral range (390–1000 nm). The wavemeter is connected to a computer by a CAMAC interface. In the design of Fischer et al. the computing time for obtaining an absolute wavelength is 20 s. The improvements in data transfer and data reduction described here reduce the computing time to less than 100 ms on a rather slow, obsolete computer (Digital PDP11/73). One can use this system to determine the absolute wavelength of successive pulses of a laser operating at a 10-Hz repetition rate. With modern computers readout rates of as much as 250 Hz should be possible

The optical design of this wavemeter is similar to that of one built earlier for operation in the UV.¹⁵ The wavemeter described here was designed to accompany tunable dye and diode lasers for spectroscopy in the visible and IR. Previous wavemeters of this type employed etalons with dielectric coatings to obtain better finesse. While they provide better spectral analysis of the laser pulse, dielectric coatings result in a narrower wavelength range. The instrument described here uses evacuated, Zerodur spaced plates coated with protected, evaporated silver (<0.25 wave of dielectric coating). A single laser pulse of arbitrary pulse duration with a few microjoules of energy is sufficient for measurement of the absolute wavelength and observation of the mode structure of laser light in the wavelength range 390-900 nm.

The calibration of etalons that employ dielectric coatings was described in two excellent papers by Lichten.¹⁶ Because the change in phase shift with wavelength of the reflected light is greater for dielectric coatings than for metal coatings, the apparent change in plate separation with wavelength is greater. Without correction for the apparent change in plate separation, metal-film-coated mirrors should provide greater wavelength accuracy. After final absolute calibration of the etalons, our error in measured plate separation as a function of wavelength is similar to that obtained by Lichten for dielectric plates, with an accuracy in plate separation for all wavelengths of ± 4 nm. The absolute accuracy of the wavemeter is 2 parts in 10^8 in the center of the wavelength range, degrading to 4 parts in 10^8 at the ends of the spectrum. In our case, the precision is limited primarily by the precision of our reference wave-



Fig. 1. (a) Schematic of a Fabry–Perot etalon: d, plate separations; t, thickness of metal coating. (b) Multiple interference in the metal coatings. The metal coatings have an index of refraction $n_1 - ik_1$, and the quartz plates have index n_2 . The plates are vacuum spaced.

lengths, particularly in the blue region of the spectrum, and by systematic errors in determining the center of etalon fringes when a centroid method is used.

2. Determination of Precise Wavelength with a Fabry–Perot Etalon

A Fabry–Perot etalon¹⁷ consists of two plane-parallel, highly reflecting surfaces separated by a distance d[Fig. 1(a)]. An incident ray is multiply reflected within the gap, and with a narrow-band point source the interference fringes are concentric rings. The working relation is

$$2d = (m + \varepsilon)\lambda,\tag{1}$$

where λ is the wavelength to be measured. The interference order m is the total number of half-wavelengths that can be accommodated between the etalon plates and is therefore a perfect integer. ε is the fractional fringe order ($0 \leq \varepsilon < 1$). For an evacuated etalon the diameter D_p of pth ring is related 18 to ε by

$$D_p^2 = m_s(p+\varepsilon), \qquad p = 0, 1, 2, \dots,$$
 (2)

where $m_s = 4f^2\lambda/d$ and f is the focal length of collecting lens. A least-squares fit to a plot of D_p^2 versus p of the measured ring diameters determines ε :

$$\varepsilon = \frac{\text{intercept}}{\text{slope}}.$$
 (3)

In the above discussion we assumed that the thickness of the reflective metallic layer is zero. For precision measurements this assumption leads to erroneous results. Thus the thickness of the metallic layer, although it is small, needs to be considered. We then see that a given partial beam is reflected back and forth not only within the medium between the two metallic layers but also within the metallic layers [Fig. 1(b)].

Taking the metal layer thickness into account, we consider a boundary value problem at each reflection in which light travels a path consisting of three media [Fig. 1(b)]: the central medium with refractive index n_0 , a metal film of thickness t and complex refractive index $n_1 - ik_1$, and a fused-silica plate with refractive index n_2 . At each interface, part of the incident light is reflected and part is transmitted. Heavens¹⁹ reviewed methods for solving the interference of light from multiple layers including metallic coatings. In Subsection 3.B we compare calculations with experiment.

Using a Fabry–Perot etalon, we determine wavelength λ from its approximately known value λ_i by the relation

$$\lambda = \frac{2d}{(m+\varepsilon)},\tag{4}$$

where 2d is measured by calibration (see Section 5 below); ε , corresponding to λ , is measured according to Eq. (3); and *m* is determined by rounding as

$$m = \operatorname{round}\left(\frac{2d}{\lambda_i} - \varepsilon\right).$$
 (5)

This calculation leads to the correct m if

$$|\Delta m| = \left| \frac{\Delta 2d}{\lambda} + \frac{(\Delta \lambda_i)}{\mathrm{FSR}} + \Delta \varepsilon \right| < 0.5,$$
 (6)

where Δm is the error in calculation of m, $\Delta 2d$ is the error in the knowledge of the etalon thickness, $\Delta \varepsilon$ is the error in the measurement of ε , and FSR is the free spectral range. Inequality (6) imposes a limit on the acceptable error $\Delta \lambda_i$ in the knowledge of λ_i . Assuming for the moment that $\Delta 2d \approx 0$ and $\Delta \varepsilon \approx 0$, the maximum permitted error in λ_i to yield the correct m is

$$(\Delta \lambda_i)_{\rm max} < {\rm FSR}/2. \tag{7}$$

[As we described below, we find in practice that statistical and reproducibility errors $\Delta \epsilon$ are negligible in inequality (7)]. Fractional errors in absolute calibration are $\Delta 2d/\lambda < 0.02$ for all etalons.]

A wavelength can therefore be measured absolutely with an etalon only if it is already known with

an error of less than one half of its FSR (= $\lambda^2/2d$) of the measuring etalon. Each etalon refines the wavelength from its less accurately known value λ_i to a value known with a precision $FSR(\Delta \epsilon/\epsilon) \approx FSR/fi$ nesse/20 of the etalon employed. This improvement over the standard resolution of an etalon comes from determining the centroid of the etalon fringe. One can therefore measure a wavelength with high accuracy by balancing the choice of the length of an etalon (i.e., the size of its FSR) with the required $(\Delta \lambda_i)_{\text{max}}$. λ_i as measured by a commonly available laboratory monochromator is not sufficiently accurate to permit the determination of a wavelength's order m for an etalon that is long enough for accuracy of 10^{-8} . Therefore we first improve the wavelength measured by a monochromator to acceptable accuracy by using two intermediate etalons. In our system the wavelength measured by the monochromator and the three etalons of the wavemeter (at 600 nm) are accurate to within ± 0.1 nm, ± 0.02 nm, ± 0.2 pm, and ± 0.009 pm, respectively.

3. Apparent Thickness of the Etalon

Bennett²⁰ described how the etalon fringe pattern changes for reflection from metal films with phase shift angles less than π :

$$(N+1+\varepsilon)\lambda = 2n_0d + (\pi-\beta)\lambda/\pi.$$
 (8)

The angle $\delta = \pi - \beta$ decreases approximately linearly as the wavelength increases, providing a nearly constant correction to the etalon separation. As a result, the method of exact fractions can be used to determine a set of integers that gives a constant $(m + \varepsilon)\lambda$ product. The dispersion in $\lambda\delta/\pi$ causes a small change in the apparent separation of the etalon plates over a broad wavelength range.

A calculation of the expected phase shift was first published by Bauer.²¹ He wrote Maxwell's equations for electric and magnetic fields in the different media illustrated in Fig. 1. Using the boundary condition that tangential components of **E** and **H** are continuous at the interface between two media, one can solve for the intensity I_{exit} of the exiting beam as a function of the actual physical separation d between the etalon plates. The phase shift δ is then calculated:

$$\delta = -\tan^{-1} \underbrace{\left(\left\{ \left[\rho^{-1/2} \exp\left(\frac{4\pi t k_1}{\lambda}\right) - \rho^{1/2} \exp\left(\frac{-4\pi t k_1}{\lambda}\right) \right] \sin(\vartheta_1 - \vartheta_2) \right\} - \left[(R^{-1/2} - R^{1/2}) \sin\left(\psi_1 - \psi_2 + \frac{4\pi t n_1}{\lambda}\right) \right] \right)}_{\left(\left\{ \left[\rho^{-1/2} \exp\left(\frac{4\pi t k_1}{\lambda}\right) + \rho^{1/2} \exp\left(\frac{-4\pi t k_1}{\lambda}\right) \right] \cos(\vartheta_1 - \vartheta_2) \right\} - \left[(R^{-1/2} + R^{1/2}) \cos\left(\psi_1 - \psi_2 + \frac{4\pi t n_1}{\lambda}\right) \right] \right)},$$
(9)

where

$$\vartheta_1 \equiv \tan^{-1} \left(\frac{-k_1}{n_1 + 1} \right), \qquad \vartheta_2 \equiv \tan^{-1} \left(\frac{-k_1}{n_1 - 1} \right), \quad (10)$$

$$\psi_1 \equiv \tan^{-1} \left(\frac{-k_1}{n_1 + n_2} \right), \quad \psi_2 \equiv \tan^{-1} \left(\frac{-k_1}{n_1 - n_2} \right), \quad (11)$$

$$\rho = \frac{(n_1 - n_2)^2 + k_1^2}{(n_1 + n_2)^2 + k_1^2},$$
(12)

$$R = \frac{(n_1 - 1)^2 + k_1^2}{(n_1 + 1)^2 + k_1^2},$$
(13)

 n_1 and k_1 are the real and imaginary parts, respectively, of the index for the metal film, and n_2 is the index for the plates.

Heavens²² reviewed several methods for solving this boundary problem. One powerful approach is the matrix method for multiple-beam interference.²³ Using the matrix approach, Heavens evaluated the complex reflectivity of the metal film [Eq. [4(140)] of Ref. 19]. Bennett published as Eq. (5) of Ref. 20 an explicit expression derived from Heavens's Eq. [4(140)] for the phase angle of the reflected wave. This complex expression appears to be quite different from Bauer's solution, written above. With modern programs for algebra, such as MATHEMATICA,²⁴ it is straightforward to compare these expressions. Using the complex math capability of MATHEMATICA, one can also determine the phase angle directly from Eq. [4(140)] of Heavens's book.

A simpler and more direct approach to calculating the phase shift is to write the interface and propagation matrices directly with MATHEMATICA. For the vacuum-metal and metal-quartz interfaces we have the equations

$$\tilde{I}_{01} = \frac{1}{\tau_{01}} \begin{bmatrix} 1 & \rho_{01} \\ \rho_{01} & 1 \end{bmatrix}, \qquad \tilde{I}_{12} = \frac{1}{\tau_{12}} \begin{bmatrix} 1 & \rho_{12} \\ \rho_{12} & 1 \end{bmatrix}, \quad (14)$$

where the transmission and reflection coefficients have the respective forms

$$\begin{aligned} \tau_{ij} &= \frac{2(n_i - ik_i)}{(n_i - ik_i) + (n_j - ik_j)}, \end{aligned} \tag{15} \\ \rho_{ij} &= \frac{(n_i - ik_i) - (n_j - ik_j)}{(n_i - ik_i) + (n_i - ik_i)}. \end{aligned}$$

Propagation within the metal film uses the matrix

$$L_{12} = \begin{cases} \exp[i2\pi(n_1 - ik_1)t/\lambda] & 0\\ 0 & \exp[-i2\pi(n_1 - ik_1)t/\lambda] \end{cases} \end{cases}$$
(16)

The matrix that describes forward and backward propagation though the multilayer stack can then be



Fig. 2. Comparison of calculated thickness of the metal coatings (solid curve) with that measured by Bennett.²⁰ For the wavelength range from 450 to 550 nm the curves are parallel but displaced by \sim 18 nm; for wavelengths greater than 550 the curves diverge.

written directly as a product of the individual matrices:

$$\tilde{S}_{02} = \tilde{I}_{01} \otimes \tilde{L}_{12} \otimes \tilde{I}_{12}. \tag{17}$$

The complex reflectivity from the film is given by the ratio of elements of this matrix:

$$\mathbf{R}_{02} = S_{21} / S_{11}. \tag{18}$$

The phase shift of the reflected ray is obtained simply in MATHEMATICA by the function

$$\beta = \operatorname{Arg}(\mathbf{R}_{02}). \tag{19}$$

MATHEMATICA retains the results as algebraic expressions similar to those of Bauer and Bennett. We simply evaluated as a functions of wavelength the phase shift from each of the four expressions for a silver film. Surprisingly, given the opportunity for a typographical error in such complicated expressions, we obtained identical results from all four. The results for a silver film with thickness t = 43.5 nm are shown in Fig. 2.

A. Complex Refractive Index of Silver

To calculate the change in phase shift with wavelength one must know the real and the imaginary parts of the refractive indices of the materials as a function of wavelength. The difficulty in measuring the dielectric properties of thin metal films was thoroughly reviewed by Heavens.²⁵ For thin films of silver with thickness less than 20 nm the phase shift and reflectivity differ dramatically from theory, which uses refractive indices measured from bulk silver. In addition, the phase shift and reflectivity vary rapidly over this range of thickness. This variation of the reflectivity from theory is a function of film growth time, which has been shown to influence the grain size of the silver films. For etalons with reflectivity greater than 90% the film has a thickness sufficient to allow it to behave as a bulk film, and the

Table 1. Wavelength Dependence of Materials in the Form $C_0 + C_1 \times X + \ldots + C_9 \times X_9^a$

Coefficient	Real n , Silver	Imaginary n , Silver	Real n , Fused Silica
C_0	-4.74527069 imes10	-2.6040170	2.71861360
$\tilde{C_1}$	$5.81882525 imes 10^{-1}$	$1.38779003 imes 10^{-2}$	-1.47281462
C_2	$-3.04231878 imes 10^{-2}$	$-6.4616631~ imes 10^{-6}$	$7.63154011 \times 10^{-5}$
$\overline{C_3}$	$8.96226071 imes10^{-6}$	$2.03531367 imes 10^{-9}$	$-2.24769812 imes10^{-7}$
C_4	$-1.64467958 imes10^{-8}$	_	$4.10725912 imes 10^{-10}$
C_5	$1.95503169 \times 10^{-11}$	_	$-4.81798763 imes10^{-13}$
C_6	$-1.50878872 imes10^{-14}$	_	$3.63045964 imes 10^{-16}$
C_7	$7.3045856~ imes 10^{-18}$	_	$-1.69774619 imes10^{-19}$
C_8	$-2.01666449 imes 10^{-21}$	_	$4.4816455~ imes 10^{-23}$
C_9	$2.42280158 imes 10^{-25}$	_	$-5.10083916 imes 10^{-27}$
R	0.9977	0.9999	0.9997

^aData for fused silica were taken from Ref. 28.

refractive index of the bulk metal accurately predicts the reflectivity.

Silver data were taken from the Handbook of Optical Constants of Solids,²⁶ in which the data over the spectral range 360-2000 nm were taken from measurements by Winsemius *et al.*²⁷ The imaginary part of the index of refraction *k* could be fitted with a third-order polynomial with parameters given in Table 1. The fit is accurate to 0.2% for wavelengths from 390 to 1500 nm. The real index *n* had a more complicated wavelength dependence and was fitted with an eighth-order polynomial, again with the parameters given in Table 1. The function fits the data within $\pm 5\%$ throughout the wavelength range, the error caused by scatter in the data.

B. Comparison with Experiment

Bennett²⁰ measured the absolute phase shift β for reflectivity of vapor-phase-grown aluminum and silver films. For films of silver of 43.5-nm thickness she reported measurements of β as a function of wavelength from 450 to 600 nm. The calculated phase shifts as described above have a similar change in phase with wavelength, but the theoretical phase shifts are uniformly 5° larger than the measurements. Bennett stated that uniformity in film thickness was essential for accurate measurement of the absolute phase shift. She determined that thickness variations were less than 0.5 nm for her silver films, with a similar error in known thickness. We found that a change from 44 to 54 nm in thickness tof the metal film in the model caused a change in phase shift of less than 1° over the full bandwidth from 400 to 900 nm. Inasmuch as etalons are insensitive to the fixed absolute phase shift of the film, only the change in apparent thickness with wavelength is important to their calibration. We converted the phase shift to the change in plate separation $\lambda\delta/\pi$ for both Bennett's data and the theory, and these are compared in Fig. 2. The uniform 5° increase in angle is reflected in the 15-nm shift in the theoretical thickness curve relative to the measurements that is shown in the figure. The curves are of nearly constant separation for 440-530 nm, but for longer wavelengths the data increase in thickness while the

model continues to decrease. Lichten¹⁶ numerically evaluated Eq. (5) of Bennett's paper and, in contrast to our comparison for silver, he obtained good agreement with her experimental data for aluminum thin films. For silver, the refractive-index data of Winsemius *et al.*²⁷ were measured for bulk metal rather than for evaporated films; beyond this difference we can offer no explanation for the disagreement between Bennett's data and theory.

Given the disagreement between Bennett's data and the theory, we believed that we could not rely on theory in calibrating the dispersion in apparent thickness for our etalons. We did, however, calculate it for a comparison with our calibration data. The manufacturer measured 92% reflectivity for our etalon plates at 550 nm. We calculated the reflectivity as a function of thickness at 550 nm and determined that our silver coatings would theoretically have a thickness of 50 nm. The change in apparent thickness was then calculated and compared with that measured in the etalon calibrations.

4. Wavemeter Design

A. Optics and Electronics

The design for the wavemeter, shown schematically in Fig. 3, uses three etalons, E1, E2, and E3, with increasing separation between their plates of 0.01, 0.25, and 5.0 cm, respectively. Evaporated films of silver form the reflecting surfaces of the etalon plates. (The etalons were purchased from IC Optical, Ltd., London.) The plates are polished to $\lambda/100$; hence the etalon optical finesse of ≈ 25 is limited by the reflectivity of the metal coatings. The plates of the etalons are in optical contact with Zerodur spacers. The thermal expansion coefficient of Zerodur is $\approx 5 \times$ 10^{-8} /°C. To further reduce thermal drift we mount the etalons in a temperature-stabilized chamber. The temperature inside the chamber is stable to within ± 0.02 °C. To maintain vacuum between the etalon plates we evacuate the chamber with liquidnitrogen-cooled molecular sieves. Light whose wavelength is to be measured is sent to the wavemeter through input guiding apertures A1 and A2. 50/50 beam splitter BS1 divides the incoming beam:



Fig. 3. Schematic diagram of the wavemeter. All the imaging lenses after the etalons are achromats. M's, mirrors; L's, lenses, other abbreviations are defined in text.

One beam is directed to the thickest etalon (E3) and the other is further divided by another 50/50 beam splitter (BS2). These beams are directed to the medium-spaced etalon (E2) and the thin etalon (E1). Cylindrical lenses CL1 and CL2 before the etalons provide horizontal wedges of light. Interference fringes produced by the thin and the medium etalons, respectively, are collected by achromatic lenses L1 and L2. Telescope T collects the fringes from the thickest etalon. The focal lengths of the lenses and their locations are adjusted such that the resultant circular fringes fall on horizontally linear diode arrays D1-D3 (EG&G Reticon Model 1024g), which are mounted upon adjustable vertical positioners. The diode heights are 432 µm, so only a chord though the ring pattern is recorded. A computer program is used to read the diameters of the measured rings continuously; adjusting an array vertically can bring the chord into coincidence with the diameter of the rings. The magnification of the optics and the number (1024) of diode elements were chosen such that three rings could be observed from the etalon and still approximately eight diode elements per fringe could be illuminated. This setup permits reasonable determination of the shape of the fringe so the centroid position of the fringe can be measured to a precision better than the diode spacing.

A computer controls the readout of the diode arrays through a CAMAC interface. A four-channel, 10-bit transient digitizer (LeCroy Model 8210) processes the video signal from each diode array, digitizing all the video signals simultaneously. The diode arrays can then be clocked in parallel by use of the clock output of this digitizer. Digitized data are stored in a memory module (LeCroy Model 8800A) with a capacity of 8096 data points per channel. The data is read out through the memory control circuitry on the digitizer. The diode arrays are controlled by sample-and-hold video circuits provided by Reticon. Each digitizer sample

then corresponds to the charge from a single diodeelement. The diode arrays and the transient analyzer are operated continuously. The video clock transitions are counted with a latching scaler (LeCroy Model 8590). At the end of each diode array scan (a frame) a pulse from the diode arrays is used to clear the scaler. When the laser fires, a trigger is provided to the wave-form digitizer that begins storing the video signal; at the same time, the trigger latches the scaler, thus recording the address within a camera frame that is being read when the laser fires. All diodes with addresses (i.e., positions within the image) earlier than when the laser fires will be dark and are not stored. All diodes with addresses after the laser fires will have been illuminated and will be stored at the beginning of the output of the transient analyzer. The illuminated diodes with addresses before the laser trigger will be read out in the next video frame and stored at the end of the analyzer array. Then the data read from the analyzer will have their left and right sides exchanged about the address of the laser trigger. The diode arrays can be read at dwell times as short as 2 µs per diode element; hence it takes a minimum of 2 ms to store 1024 values from four diode arrays (an additional array can be used with a small spectrometer). When measuring cw lasers one can change the sensitivity of the instrument by altering the dwell time. The computer (Digital Model PDP11/73) uses a crate controller (Kinetic System Model 3912) to read and control the modules. The same crate controller and computer are also used to control the lasers and digitize signal detectors for laser spectroscopy experiments.

B. Computation Procedure

The computation is done with a large program that reads the Reticon diode arrays, displays the fringes on a scope, measures the etalon ring positions, and then uses these data to compute the wavelength to be displayed on the computer terminal. The entire computation, which includes data acquisition and subsequent data analysis, takes less than 100 ms; the wavelength computation is completed within the interpulse period of a pulsed laser operating at a repetition rate of 10 Hz.

The first step in the data acquisition is the reading of the raw data. Then, for a complete picture of the exposed image, the early channels read from the second frame must be shifted relative to the late channels read from the first frame. The image is spliced at the channel read from the latching scaler. An assembly language subroutine reads the arrays from the digitizer and the channel number from the scaler and then reconstructs the correct 1024-channel image from parts of the two Reticon frames. The reading time per array is ${\approx}1.5\,\text{ms}.$ Thus the total time to read three diode arrays is \approx 4.5 ms. This process cannot be substantially sped up with a faster computer because of the 1.0-µs bus cycle time for the CAMAC. For faster readout, the CAMAC would have to be abandoned and separate digitizers and processors for analyzing each array employed.

Once the raw data ring pattern has been con-



Fig. 4. Positioning of the etalon fringes on the diode arrays.

structed, a second assembly language subroutine locates the fringes and calculates the centroid of each fringe. The magnification of the optics and the resolution of the diode arrays were chosen such that several fringes can be observed and still have several pixels distributed over the width of each peak. Thus one can calculate the centroid of the peak with a precision that is a fraction of the pixel width and compute the wavelength with an error that is less than the optical resolution of the etalon. Values of centroids are the data on which rest of the computation is based. Thus a total of 1024 data points/etalon are reduced to the number of centroids. This reduction of data increases the speed of the computation.

Once the peaks have been located and their centroids have been determined, the next step is to coordinate those pairs of peaks that constitute an interference ring. To understand the coordinating procedure we define the past known value of the common center (Fig. 4) of the ring system as the historic center (Hcent). It provides the beginning of the process for coordination of the peaks. The peaks are numbered from left to right. The peak numbers on the immediate left-right of Hcent are denoted Cleft-Cright. Two innermost rings are said to be found if

$$CC_0 - CC_1 < \text{DIFFMAX},$$
 (20)

where

$$CC_0 = [\text{Centroid}(\text{Cleft}) + \text{Centroid}(\text{Cright})]/2,$$

 $CC_1 = [\text{Centroid}(\text{Cleft} - 1) + \text{Centroid}(\text{Cright} + 1)]/2,$
(21)

and DIFFMAX is the maximum permitted difference of the centers of two rings. The exact value of this maximum permitted difference is based on the accuracy with which we can measure the centroids. In our system DIFFMAX is set to 2 pixels. Note that if a fringe of a ring is missing because it is not properly illuminated or because its centroid is shifted by noise, the program will obtain a fringe on that side from the next outer ring. The center from this ring will then be different from the center from the other ring. The program is designed to recover from this failure by trying all possible coordinations of peaks to obtain as many rings with common centers as possible. The program also numbers each complete ring. A ring with a missing peak is skipped in the sequence, but the program can still measure the fraction order ε for that set of rings.

The established common center of ring system is

$$CC = (CC_0 + CC_1)/2.$$
(22)

It is this common center that is used to coordinate third and subsequent outer rings. It is also this center that is preserved in a running average as Hcent for later use.

The *p*th outer ring has been found if

$$CC_p - CC < \text{DIFFMAX},$$
 (23)

where

$$CC_p = \{\text{Centroid}[(\text{Cleft} - 1) - (p - 1)] + \text{Centroid}[(\text{Cright} + 1) + (p + 1)]\}/2 \quad (24)$$

and p = 0, 1, 2, ..., N.

The measured diameter of the *p*th ring in terms of the number of pixels is given as

$$D_{p} = [\text{Centroid}(\text{Cright} + p) - \text{Centroid}(\text{Cleft} - p)]$$
(25)

An (N + 1)-point least-squares fit to $D_p^2 | p$ determines [Eq. (2)] the fractional fringe order ε . The order of interference is then determined from Eq. (5), and the improved value of the wavelength is given by Eq. (4).

The above procedure is repeated three times, first for the thin etalon (E1), then for the medium etalon (E2), and finally for the thickest etalon (E3); each time the wavelength refined by the previous etalon determines the order of the next.

5. Calibration

Any unknown wavelength can be determined from the measured fractional fringe order, provided that the etalon's thickness is known with sufficient accuracy. An accurate determination of each etalon's thickness is required for calibration of the wavemeter. The calibration is done with reference wavelengths. A given laser line can be used as a reference, provided that it has been measured with an accuracy equal to or better than the accuracy of the etalon to be calibrated; such measurements should be done with respect to an internationally accepted standard. Further, the reference laser should be frequency stabilized with stability better than the accuracy of the etalon to be calibrated. The accuracy of our final etalon of the wavemeter is of the order of 3×10^{-8} , but the wavelengths of commonly available laboratory lasers are not known with such high accuracy. This lack of precision presents a serious problem that is further complicated by the apparent change in thickness of the etalons with changes in wavelength.

A. Method of Exact Fractions

Most calibration methods are based on the method of exact fractions.¹⁷ Using the etalon equation

$$(m+\varepsilon)\lambda = 2\left\{d - 2\left[s_0 - \frac{\mathrm{d}s}{\mathrm{d}\lambda}\left(\lambda - \lambda_0\right)\right]\right\}$$
 (26)

and given two wavelengths, one precisely known (λ_0) and another approximately known (λ_1) , which are sufficiently close in wavelength that one can neglect the change in separation of the plates with wavelength ds/d λ , one obtains two equations:

$$(m_0 + \varepsilon_0)\lambda_0 = 2d_0 \tag{27a}$$

$$(m_1 + \varepsilon_1)\lambda_1 = 2d_0, \tag{27b}$$

where $d_0 = d - 2s_0$ is the calibrated plate separation at λ_0 . [In terms of the previous discussion of phase shift, $s_0 [-(ds/d\lambda)](\lambda - \lambda_0)$ is a first-order Taylor expansion of $\lambda\delta/2\pi$.] Now there are three unknowns, the two integer orders and the separation. One can use a bootstrap technique to measure λ_1 precisely.⁶ Solving Eqs. (27) for the order and subtracting, one can obtain an estimate for the separation:

$$2d_0 = \frac{\Delta m + \Delta \varepsilon}{\left(\frac{1}{\lambda_0} - \frac{1}{\lambda_1}\right)},\tag{28}$$

given an independent measurement for the change in order Δm . One can substitute this separation into Eq. (27a) to calculate the order m_0 :

$$m_0 = \text{Round}(2d_0/\lambda_0 - \varepsilon_0). \tag{29}$$

Finally, the wavelength λ_1 can be determined with precision $\delta \varepsilon / m_0$:

$$\frac{\lambda_1}{\lambda_0} = \frac{(m_0 + \Delta m + \varepsilon_1)}{(m_0 + \varepsilon_0)}.$$
(30)

If one uses a tunable laser, one can measure Δm by scanning from the known wavelength λ_0 to λ_1 while counting mode hops of the etalon. In addition to having the wavelengths' close enough that the change in apparent separation can be neglected, one must estimate λ_1 to a precision of $1/(4 m_0)$. For the thin etalon, measurements with a monochromator are sufficient. For the etalons with greater spacing, the wavelengths can be estimated with the nextthicker etalon. For calibrations over broad wavelength ranges, this method is limited in accuracy by the change in apparent separation with wavelength.

We had hoped to use theoretical calculations of the change in etalon separation to improve the calibration. Given the disagreement between theory and experiment for the phase shift, this was not possible. The change in thickness would have to be measured directly with a series of known calibration sources spread over the visible spectrum. Because all our etalon plates were coated at the same time, we planned to measure the change in thickness of the thinnest etalon where the change would have the greatest effect. This would require references with less precision than the wider-spaced etalons.

Inasmuch as most of the reference sources were fixed in wavelength, we could not measure Δm by independent means. We then used a variation of the method of exact fractions. First we used the manufacturer's specified separation d to estimate the order, using Eq. (29) at the primary reference $\lambda_0 (m_{\rm app} \sim 269$ for the thin etalon). Then, for all integer orders $m_{\rm app} + (\pm i)$ over a range corresponding to twice the manufacturer's specified error in separation (250 < m < 400 for the thin etalon), we calculated the thicknesses at λ_0 , using Eq. (27a). For each of these (2i + 1) trial thicknesses,

$$2d_i^{(\text{trial})} = \{ [m_1^{(\text{app})} + (\pm i)] + \varepsilon_1 \} \lambda_1, \quad i = 0, 1, 2, 3, \dots,$$
(31)

we calculated an order for each reference wavelength:

$$m_{k} = \left[\frac{2t_{i}^{(\text{trial})}}{\lambda_{k}} - \varepsilon_{k}\right].$$
(32)

The different values of m_k will in general be different from an integer, but at the correct value for m_0 all m_k will be close to an integer. Choosing m_0 as the correct one, we round all the corresponding orders m_k at the reference wavelengths and calculate the corresponding separation from Eq. (27b). This process is rapidly done with a spreadsheet program.

B. Raman Method

In Raman-induced Kerr-effect spectroscopy²⁹ and other similar four-wave mixing experiments the line frequency (ν_R) of a Raman line of a molecule, the pump frequency (ν_{pump}), and probe frequency (ν_{probe}) at resonance are connected by the equation

$$\nu_R = |\nu_{\text{pump}} - \nu_{\text{probe}}|. \tag{33}$$

If ν_{R_1} and ν_{R_2} denote the line positions of two Raman lines and $\nu_{\text{probe}}^{(1)}$ and $\nu_{\text{probe}}^{(2)}$ are their respective resonance probe frequencies, then for a fixed-frequency pump and a tunable probe

$$\nu_{\text{probe}}^{(1)} - \nu_{\text{probe}}^{(2)} = \nu_{R_2} - \nu_{R_1}.$$
 (34)

If ε_1 is the fractional fringe order of $\nu_{\text{probe}}^{(1)}$ and ε_2 that of $\nu_{\text{probe}}^{(2)}$ in the etalon to be calibrated, there are N complete FSR's of the etalon within the frequency span $\nu_{\text{probe}}^{(12)} = [\nu_{\text{probe}}^{(1)} - \nu_{\text{probe}}^{(2)}]$:

$$(1 - \varepsilon_1)$$
FSR + N (FSR) + ε_2 (FSR) = $(\nu_{R_2} - \nu_{R_1})$. (35)

The FSR of the etalon to be calibrated is then given by

FSR =
$$\frac{(\nu_{R_2} - \nu_{R_1})}{(\varepsilon_2 - \varepsilon_1) + (N+1)}$$
. (36)

This suggests a calibration method: Select two (or more, if necessary) narrow Raman lines with known frequencies that are separated by several FSR's of the

etalon to be calibrated and that occur within the tuning range of the probe laser. Tune the probe laser to the first Raman line, and measure the fractional fringe order. Then, while observing the fringes on an oscilloscope, tune in small (less than one FSR) steps toward the second Raman line. Count the number of times that the fringe pattern shows an intense peak at the center of the observed fringes, corresponding to $\varepsilon = 0$. The total number of complete FSR's between the two lines is then given by N = n - 1. Finally, measure the fractional fringe order at the frequency of the second Raman line. The FSR and hence the calibrated thickness of the etalon are then calculated from Eq. (36). Because N is exact, the only uncertainties in the calculation are the uncertainty in the measurement of fractional fringe orders and the uncertainty in the Raman frequencies.

C. Calibration Sources

An intracavity, iodine-stabilized helium-neon laser operating at 633 nm is an excellent standard of wavelength.³⁰ This laser was originally designed by Layer.³¹ In this laser the accuracy of laser lines, locked on hyperfine transitions of iodine, is a few parts in 10¹⁰. For the purpose of calibration of this wavemeter, we duplicated Layer's design in our laboratory and locked the laser on to the *i* hyperfine component of the iodine $(^{127}I_2)$ absorption line. The absolute wavelength of this line is $632\ 991.398\ \pm\ 0.003\ pm.^{32}$ Another calibration line was obtained from a semiconductor diode laser stabilized to the hyperfine transition $[6P_{3/2}^2(F=3) \leftarrow 6S_{1/2}^2(F=3)]$ of atomic cesium, whose wavelength was measured with an accuracy of few parts in 10⁸ with respect to an iodinestabilized laser.^{33,34} The rubidium lines were measured with a Coherent 899 Ti:sapphire laser and Doppler-free saturation spectroscopy. The frequency of these lines has been measured within an accuracy of $3 \times 10^{-9.35}$

Some additional, but less accurately known, calibration wavelengths were obtained from uranium emission spectra in a hollow-cathode discharge lamp. Because uranium is a heavy element and the Doppler broadening for a given temperature varies inversely as the square root of the atomic weight, these lines are narrow. There are several lines^{36,37} distributed throughout the wavelength range $\approx 385 - \approx 909$ nm. These lines are known within an accuracy of 10^{-7} , which is sufficient for all but the thickest etalons. As a pulsed dye laser is tuned, uranium resonances in the discharge are observed by the optical galvanic effect.^{38,39}

For calibration of the thin etalon, several lines of an argon-ion laser and the line of a green helium– neon laser were used. These lines are known to an accuracy of ± 0.02 nm.⁴⁰ We were concerned that differences in magnetic fields for plasma confinement and in gas pressures in our lasers might shift the wavelengths for these sources. Once the etalon with 2.5-mm spacing was calibrated, we could measure the

Table 2.	Reference	Lines	Used for	Calibration

	Wavelength		
Reference Line	(nm)	This Work	
Fourth anti-Stokes of YAG	410.8300(4)	_	
Third anti-Stokes of YAG	435.6778(3)	_	
Second anti-Stokes of YAG	463.7247(3)	-	
Ar^+ 476	$476.622(6)^a$	476.614(2)	
Ar^+ 488	$488.122(6)^a$	488.118(2)	
First anti-Stokes of YAG	495.6311(3)	-	
Ar^+ 497	$496.645(4)^a$	496.6412(1)	
Ar^+ 514	$514.676(4)^a$	514.6877(1)	
He–Ne green	543.5159(5)	543.5213(1)	
Ne	$572.094(4)^{b}$	572.0935(2)	
First Stokes of YAG	574.7177(2)	574.6745(1)	
U	$576.20331(5)^b$	-	
Second Stokes of YAG	624.5463(2)	_	
He–Ne red	$632.991398(3)^c$	_	
$\operatorname{Rb} D_2 \mathrm{d/f}$	$780.2462916(8)^d$	-	
$\operatorname{Rb} D_2 \mathrm{b/f}$	$780.246450(2)^d$	_	
$\operatorname{Rb} D_1$ c'	$794.981364(2)^d$	_	
$\operatorname{Rb} D_1 \operatorname{d}'$	$794.974964(3)^d$	-	
Cs	$852.33512(4)^e$	-	

^aRef. 40.

^bRef. 36.

^cRefs. 30 and 31. ^dRef. 35.

^eRefs. 33 and 34.

wavelengths for our particular laser to a precision of

 5×10^{-7} . A set of more-precise references was generated by Raman shifting of a frequency-doubled, narrow band (±50-MHz) YAG laser in CO₂ at 1 amagat, permitting us to measure two Stokes and four anti-Stokes lines from the 1388-cm⁻¹ Raman resonance. This resonance has a width less than 0.04 cm⁻¹ at 2 amagats.⁴¹ The centroid of this resonance was measured by Sussmann *et al.*⁴² as

$$\nu_R = 1388.177(5) - 0.71(10) \times 10^{-2}P,$$
 (37)

with ν_R in inverse centimeters and P in amagats. The second Stokes line is near our primary reference (red iodine-stabilized helium-neon laser). In fact the first and second Stokes lines bracket the wavelength of the helium-neon laser and are sufficiently close in wavelength that the change in thickness can be ignored and the Raman shift measured directly. We measured 1388.223 cm⁻¹; the difference compared with Eq. (37) may result from laser-induced shifts. We used the measured shift to calibrate the wavelengths of the anti-Stokes transitions. The use of these references is described in Subsection 5.D. The reference sources and wavelengths are listed in Table 2.

D. Results

1. Etalon 1

The thin etalon was calibrated by the modified method of exact fractions as described in Subsection 5.A. A section of the iteration table for etalon 1 on

Table 3. Section of Iteration Table for Etalon 1

m_0 He–Ne	$2d_0$	\mathbf{Cs}	YAG Second Stokes	U	YAG First Stokes	He–Ne Green
632.991395		852.33512	624.49286	576.20331	574.6745	543.5159
:		:	÷	:	:	:
326	2065944.6	242.33854	329.929547	358.488592	358.481367	379.153492
327	2072274.6	243.081195	330.943131	359.587148	359.582821	380.318116
328	2078604.5	243.823851	331.956714	360.685704	360.684275	381.482739
329	2084934.4	244.566507	332.970297	361.784259	361.785729	382.647363
330	2091264.3	245.309162	333.98388	362.882815	362.887183	383.811986
331	2097594.2	246.051818	334.997463	363.981371	363.988637	384.97661
332	2103924.1	246.794473	336.011046	365.079926	365.090091	386.141233
333	2110254.1	247.537129	337.02463	366.178482	366.191545	387.305857
334	2116584.0	248.279784	338.038213	367.277037	367.292999	388.47048
335	2122913.9	249.02244	339.051796	368.375593	368.394453	389.635104
336	2129243.8	249.765096	340.065379	369.474149	369.495907	390.799727
:	:	:	:	:	:	:

integer order m_0 is reproduced in Table 3. Note that at $m_0 = 331$ the orders are within ± 0.02 of an integer for all other reference wavelengths. This was the only order for $250 < m_0 < 400$ for which this coincidence occurs. Assigning $m_0 = 331$ and rounding the orders to the nearest integer, we can calculate the thickness at each wavelength. These thicknesses are plotted in Fig. 5 as a function of wavelength. A least-squares fit to the thickness measurements yields the solid curve in Fig. 5. The scatter observed in the data is consistent with the reproducibility in measuring ε , $\Delta \varepsilon \sim 0.005$. The error bars shown are a combination of the uncertainty in the wavelengths shown in Table 2 and the fractional error in measuring the order $\Delta \varepsilon/m$. For this etalon the latter is the dominant error.

The change in apparent thickness shown in Fig. 5 for our thin etalon is 100 nm over the wavelength range of our reference sources. This is nearly an order of magnitude greater than that expected theoretically. We believe that this large change is due to optical aberrations generated by the large angles for the outer rings in the case of the thin etalon. At the cesium wavelength the fourth ring has a half-



Fig. 5. Calibrated plate separation for etalon 1 as a function of wavelength. The change in plate separation greatly exceeds that expected from the metal coatings and is likely caused by chromatic aberration in the optics resulting from the large angles subtended by the rings for the small $(100-\mu m)$ plate separation.

angle of 9°. We had noticed that in the case of the thin etalon the analysis for fractional fringe ε depended on the number of rings used in the fit. Equation (2) results from a small-angle approximation.^{17,18} The program was corrected to account for the angle of the ring in the fit, but the large angles could cause other aberrations, including spherical aberration of the imaging lens, change in effective focal depth because of the thick etalon plates, and the variation of metal coating thickness with angle. We assume that the apparent change in thickness observed in Fig. 5 stems from a combination of all these effects. Using the calibrated change in thickness, we determined the precision of the thin etalon of 3×10^{-5} , which is sufficient to permit us to determine unambiguously the order for etalon 2, the 2.5-mm spaced etalon.

2. Etalon 2

We used a similar procedure to analyze etalon 2, spaced by 2.5 mm. Because of the larger order for this etalon, more precision is required for the reference wavelengths. We found that measurements of the order were reproducible over long term for the best wavelength references to better than $\Delta \varepsilon = 0.002$. The procedure of iterating the integer order for the reference wavelength λ_0 was done with only the cesium wavelength, the Raman wavelengths, and the rubidium wavelengths.

We put the Raman wavelengths on an absolute scale by measuring the line produced by second Stokes scattering of the YAG laser. This wavelength lies sufficiently close to the primary reference wavelength that the change in apparent thickness can be neglected. Then, for each trial order for the primary reference wavelength, we calculated the order of the second Stokes line from

$$m_k = \operatorname{Round}\left[\frac{2d_i^{(\operatorname{trial})}}{\lambda_k} - \varepsilon_k\right],$$
 (38)



Fig. 6. Calibrated change in plate separation relative to the separation at the red helium-neon wavelength for the etalon 2, spaced by 2.5 mm. Error bars, uncertainties in the fractional fringe order and known wavelengths of the reference sources; solid curve, expected variation from theory. The reproducibility of the measurements is consistent with the error bars; hence the disagreement with theory represents a systematic error that is due to the method used for measuring the fringe or to optical abberations.

using its estimated wavelength. The wavelength was then calculated from Eq. (27b). All other Raman sources were related to the second Stokes line by

$$\frac{1}{\lambda_n} = \frac{1}{\lambda_{2nd \text{ Stokes}}} + n\nu_R, \tag{39}$$

where ν_R is given by Eq. (37). In iteration over orders ranging from 7600 to 7950, orders of reference wavelengths were near integers for $m_0 =$ 7757, 7871, 7831, 7953, 7968. For each of these possible orders the thickness for each reference wavelength was calculated and plotted. The order $m_0 =$ 7757 gave significantly smaller residual values of ε at each wavelength. It should be pointed out that before making the rubidium measurements we obtained a calibration $m_0 = 7928$, which gave residual values for ε that were smaller than those when the rubidium measurements were included, but this calibration gave significant errors at the rubidium wavelengths. This confirms the necessity for making accurate calibrations over a broad spectral range. A plot of the difference in thickness relative to the iodine-stabilized helium-neon laser wavelength for $m_0 = 7757$ is shown in Fig. 6 and compared with theory. Again each error bar is a combination of the uncertainty in measuring the order and the error in our knowledge of the reference wavelength. In addition to producing a plot closer to theory, the sum of the absolute differences of the reference orders from integers (a measure of the error) was a factor of 3 smaller for $m_0 = 7757$ than for all other possible orders.

The data of Fig. 6 show systematic differences from values expected from theory. These differences correspond to systematic errors of the order of ± 0.01 in the measurement of ϵ . These errors in absolute calibration at different wavelengths are a factor of 4 larger than the long-term (more than a year) repro-

ducibility for the measured ε at each specific wavelength. If a specific measurement is repeated months apart we observe variations of less than ± 0.002 at each wavelength; yet the difference in calibrated thickness differs from theory by a reproducible scatter corresponding to $\delta \varepsilon \sim \pm 0.01$.

In comparing the measured change in thickness with that calculated, we find that, unlike in the data of Bennett, the apparent plate separation continues to decrease for wavelengths longer than 550 nm. In fact, the data of Fig. 6 decrease faster than the model for the longer wavelengths. The experimental data and the model differ by nearly 20 nm at the larger wavelength.

We speculate that the error in measured thickness change is due either to the protective coatings over the silver or to a systematic error that results from using the centroid to measure the fringe location rather than fitting the known fringe shape to the data. Such a fit would greatly slow the speed of the wavemeter unless a moment method of determining the most probable position of the fringe could be used.

3. Etalon 3

Using the reference wavelengths in the red and the IR and the method of exact fractions, we determined a unique calibration, $m_0 = 157941$ and $2d = 99\,975905 \pm 1$ nm, at the helium-neon reference wavelength for etalon 3. The measured change in thickness with wavelength then gave agreement with the IR calibration wavelengths, with a decrease in plate separation of ± 5 nm relative to the thickness at the helium-neon wavelength. This shift is smaller than that observed for etalon 2 but confirms a systematic error in the measured phase shifts shown in Fig. 6 in that the thickness variations for the two etalons are different.

We had hoped to use the thickness calibration from etalon 2 to calibrate the thickness change for the 5.0-cm-spaced etalon. Because all etalon plates were coated in the same evaporation run, we assumed that the phase shifts would be identical for all the coatings as long as the thickness was uniform over the evaporation chamber. We had hoped to test this assumption by comparing the change in thickness for the two smaller-spaced etalons. Because we found that the thickness variations differ systematically among the three etalons, we determined that the absolute calibrations are limited not by phase shifts in the coatings but by other systematic errors. As the calibrations are reproducible, we use the fit of a smooth function to interpolate between the calibrations. Conservatively the accuracy of the instrument at the ends of the wavelength range is 4×10^{-8} . In the region from 550 to 700 nm the accuracy is 2 imes 10^{-8} .

6. Conclusions

We have presented a computer-controlled pulsed wavemeter that has a precision of 1 part in 10^8 and an absolute accuracy of a few parts in 10^8 . This wavemeter is capable of computing the wavelength of

each shot of the laser in real time at repetition frequencies up to 10 Hz. Using modern computers and parallel processing the data from each etalon with a separate CPU can enhance the speed of computation to repetition rates of 250 Hz.

The use of evacuated silver-coated etalons reduced the temperature and wavelength dependence of the refractive index between etalon plates and extended the wavelength range of the wavemeter but introduced a wavelength-dependent phase shift of the silver coating. Given the disagreement in change of thickness that we found between the experiments of Bennett²⁰ and theory, aluminum may be a better choice for coatings. Because of the lower reflectivity of aluminum films and the resultant smaller etalon finesse, however, aluminum may not yield accuracy greater than that given by the instrument reported here.

Often such wavemeters are built⁴³ but cannot be used for absolute wavelength measurement because they are not calibrated. The calibration of our instrument would have been greatly simplified if we could have used as a reference a cw, narrow-band dye laser operating in the blue–green wavelength range by measuring with a scanning cw wavemeter. As many researchers may have similar circumstances, we have described several calibration methods that are usable with only pulsed dye lasers.

Although the development of such a high-accuracy wavemeter required considerable effort, we find the device to be a useful tool for monitoring the mode structure, laser frequency, laser bandwidth, and frequency jitters of both cw and pulsed lasers.

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