Design of a static Fourier-transform spectrometer with increased field of view

J. Courtial, B. A. Patterson, A. R. Harvey, W. Sibbett, and M. J. Padgett

We present several novel designs of static Fourier-transform spectrometers based on Wollaston prisms. By numerical modeling we show the increased field of view that can be obtained when an achromatic half-wave plate is included between the prisms or when prisms fabricated from positive and negative birefringent materials are combined. In addition, we model how a single Wollaston prism with an inclined optic axis produces a fringe plane localized behind its exit face, thus enabling the design of a static Fourier-transform spectrometer based on a single Wollaston prism. © 1996 Optical Society of America

Key words: Wollaston prism, multichannel Fourier-transform spectroscopy, field of view, optical throughput, étendue.

1. Introduction

Many scientific and industrial applications require compact spectrometers with high étendue (optical throughput). The étendue E is defined as the product of the instrument aperture A and the solid-angle field of view Ω. Fourier-transform (FT) spectrometers can have a significantly higher étendue than dispersive instruments with a similar resolution (Jacquinot advantage).[1]

In a FT spectrometer the output of the interferometer is recorded as a function of the difference in length between the two optical paths, and the spectrum is given by the FT of this interferogram. The resolution in wave numbers $\Delta\sigma$ is related to the maximum path difference Δ_{max} by the equation[2] $\Delta\sigma = 1/\Delta_{max}$.

Birefringent optical components are used in the design of interferometers that form an interferogram that can be recorded with a linear detector array. These multichannel FT spectrometers have no moving parts and can be low in cost, compact, and robust.

For example, multichannel FT spectrometers based on Savart plates[3] can have a field of view that is limited only by the numerical aperture of the optical components. However, the need for a lens sets a lower limit to the size of the instrument. In addition, the interferogram in existing Savart plate interferometers is not only distorted by lens aberrations, but also by the Savart plate itself.[4] This latter distortion could be reduced by the use of a modified Savart plate.[5]

Alternatively, a number of studies have reported multichannel FT spectrometers based on Wollaston prisms.[6,7] These spectrometers produce undistorted fringe patterns and can be configured without the need for lenses,[8] giving extremely compact and simple designs. In this paper we report novel designs of Wollaston prism spectrometers that combine these advantages with significantly increased fields of view.

Wollaston prisms are made from two similarly shaped wedges of birefringent material joined along their hypotenuse to form a rectangular block. The optic axes of the two components are perpendicular to each other and parallel to the external faces of the block. Wollaston prisms are used commonly as polarizing beam splitters, introducing an angular separation between orthogonally polarized components of the incident light.

When a Wollaston prism is placed between crossed polarizers and illuminated by a point light source, nonlocalized straight-line interference fringes are observed after the second polarizer. This is due to a path difference between the two polarization components that varies linearly in the direction perpendicular to the fringes and parallel to the faces of the Wollaston prisms. If the extent of the light source is

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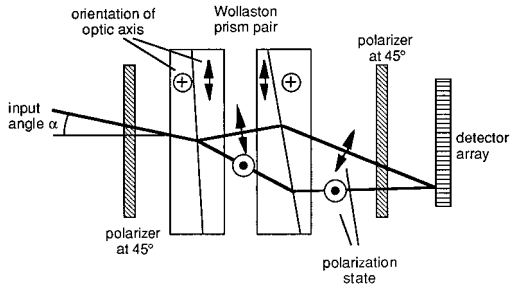

Fig. 1. Schematic layout of a static FT spectrometer based on a pair of Wollaston prisms.

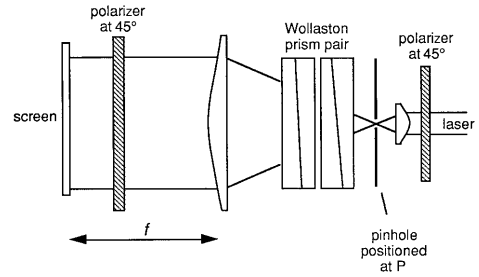

Fig. 2. Experimental arrangement for the observation of the interferogram corresponding to the field of view for a pair of Wollaston prisms.

increased, the fringes become increasingly localized around a plane, the so-called fringe plane, that lies within the prism itself. The relation between the spatial intensity distribution and the spectrum of the light source is essentially a FT.[9]

A lens can be used to image these fringes onto a detector array. Alternatively, one can make a more compact spectrometer by adding a suitable second Wollaston prism between the polarizers, which forms another fringe plane behind the second polarizer. A detector array can record the intensity distribution in the fringe plane from which the spectrum of the light source can then be calculated. A schematic layout of such an instrument is shown in Fig. 1.

In all FT spectrometers the shortest wavelength that can be identified unambiguously is determined by the Nyquist requirement to sample the interferogram at least twice per fringe. Consequently, in a static FT spectrometer the maximum path difference is limited effectively by the number of elements on the detector array. For a spectrometer operating in the visible region of the spectrum incorporating a detector array of 500–1000 elements, the maximum path difference is restricted to be of the order of 10's of micrometers. To draw a fair and practically relevant comparison between our proposed configurations we take a ± 45-μm path difference and a 6-mm detector aperture as our standard design requirement.

2. Angular Path Difference and Field of View

For an ideal design of an interferometer-based spectrometer with unlimited field of view, the difference $\Delta_P(S)$ between the two optical paths from a point source at a position S to a detector position P in the fringe plane is independent of S. For an infinitely distant light source, S can be described in terms of angles α and β relative to the optical axis, under which light from S enters the prisms. We have taken α to be the angle in the plane containing both the optical axis and the normal to the interface within the Wollaston prisms.

In reality, however, the path difference does change with angle of incidence and this limits the useful field of view. At a wavelength λ, the field of view can be defined usefully as the range of input angles α and β over which the path difference does not vary by more that $\lambda/2$, i.e., $|\Delta_P(\alpha, \beta) - \Delta_P(0, 0)| \leq \lambda/2$.

In our research we developed a ray-tracing program for birefringent media that incorporates equations derived by Simon and Echarri[10] and that is presented in MATHEMATICA.[11] With this program, the path differences $\Delta_P(\alpha, \beta)$ as functions of angles of incidence are calculated numerically for various spectrometer configurations.

The angular dependence of $\Delta_P(\alpha, \beta)$ can also be studied experimentally if one positions a highly divergent point light source, e.g., a pinhole illuminated by a laser, at P and observes the far-field interference pattern on the other side of the Wollaston prisms (see Fig. 2). In essence, the experimental arrangement shown in Fig. 2 has the pinhole placed at the position of the detector in Fig. 1 and consequently is the inverse setup of Fig. 1. The interference pattern is the result of an inverse ray trace from a point at the detector position. We refer to these interference patterns as field-of-view interferograms. The separation between adjacent fringes corresponds to a change in the path difference of one wavelength. The experimentally derived field-of-view interferograms can be compared with those predicted as a means to verify the accuracy of the model. All comparisons were based on the use of a He–Ne laser with $\lambda = 632.8$ nm.

3. Field of View of a Fourier-Transform Spectrometer Based on a Michelson Interferometer

Conventional FT spectrometers are based on Michelson interferometers. The condition on the field of view as defined above can be approximated as[12] $i \leq (2\lambda/\Delta_{\max})^{1/2}$ where i is the angle between the direction of light incidence and the optical axis and corresponds to $\sqrt{\alpha^2 + \beta^2}$ in the case of a Wollaston prism spectrometer.

For a FT spectrometer based on a Michelson interferometer, a maximum path difference of 45 μm implies a circular field of view at 633 nm of $\pm 9.6°$.

4. Field of View of Previous Fourier-Transform Spectrometers Based on Wollaston Prisms

One specific design of a Wollaston prism spectrometer[8] comprises two 4-mm-thick, 10-mm aperture calcite Wollaston prisms with internal angles of 3.6° and 5.9° producing a fringe plane 2 mm behind the exit aperture of the second prism. With a 6-mm-wide CCD detector array, the maximum path difference is ± 45 μm.

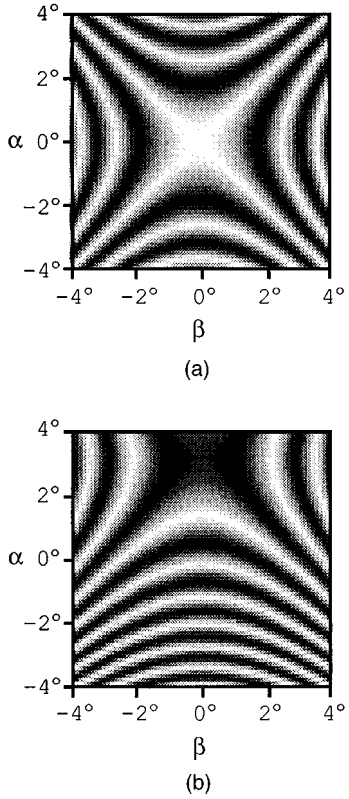

Fig. 3. Calculated field-of-view interferogram for the nonoptimized double Wollaston prism spectrometer described in Section 4. (a) P on axis in fringe plane, (b) P on axis 2.0 mm behind fringe plane.

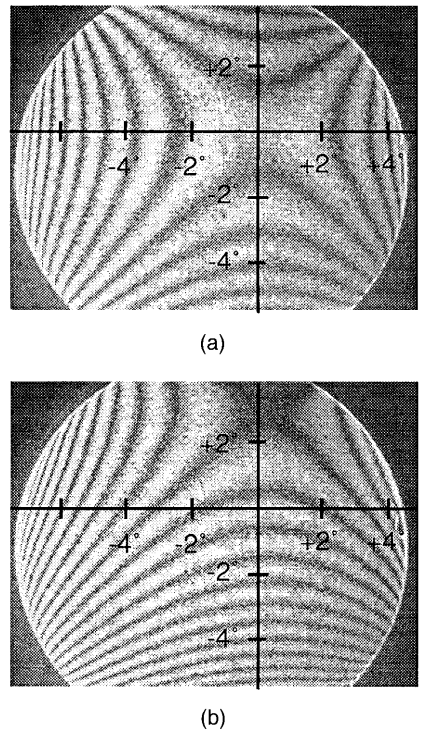

Fig. 4. Observed field-of-view interferogram for the double Wollaston prism spectrometer described in Section 4: (a) pinhole in fringe plane ±0.1 mm, (b) pinhole 2.0 ± 0.1 mm behind fringe plane.

The analytic approximation for the field of view of a combination of birefringent plates with crossed optic axes (i.e., an idealized Wollaston prism) of overall thickness $2t$ is given by[13]

$$\alpha^2 + \beta^2 \leq \frac{\lambda}{t}\left(\frac{n_o{}^2 n_e}{n_e{}^2 - n_o{}^2}\right). \tag{1}$$

Note that the field of view depends on the overall thickness of the birefringent material. At 633 nm and a total thickness for the calcite prism pair of 6 mm, this predicts an angular extent of approximately ±2°, which is smaller than that of a standard Michelson-based spectrometer with the same maximum path difference.

Figure 3 shows calculated field-of-view interferograms for the spectrometer described above. We note that fringes of constant path difference are hyperbolic. When P is positioned in the fringe plane, the fringe pattern is centered about normal incidence and the field of view is confirmed to be approximately ±2°. Further modeling shows this largely to be unchanged for the off-axis pixels. Figure 3(b) shows the importance of accurate positioning of the detector. As the detector is moved away from the fringe plane, the fringe pattern is no longer centered about normal incidence, and the field of view for an on-axis light source is reduced.

Repeated ray tracing shows that the position of the fringe plane largely is insensitive to the wavelength, moving by less than 25 μm over the operating range of the spectrometer (350–1000 nm). Consequently, for this design the optimum position of the detector array essentially is independent of the wavelength of the incident light.

To confirm the accuracy of the model, the predicted interferograms shown in Fig. 3 can be compared with the experimentally observed interferograms shown in Fig. 4. Note that the shape and scale of the predicted and observed fringes are identical.

5. Static Fourier-Transform Spectrometers Based on Wollaston Prisms with Increased Fields of View

The field of view of a Wollaston prism can be increased by the addition of a compensation plate.[13] Compensation of the angular dependence of the path difference arises if one ensures that the path difference that is introduced by the compensation plate is equal in magnitude but opposite in sign to that of the Wollaston prism. For optimum cancellation, the relative thickness of the Wollaston prism t_W and of the compensation plate t_c is given by

$$\frac{t_W}{t_c} = \left|\frac{\left(\dfrac{n_e{}^2 - n_o{}^2}{n_o{}^2 n_e}\right)_c}{\left(\dfrac{n_e{}^2 - n_o{}^2}{n_o{}^2 n_e}\right)_W}\right|. \tag{2}$$

The compensation plate can be made either of material with a different sign of birefringence than that of

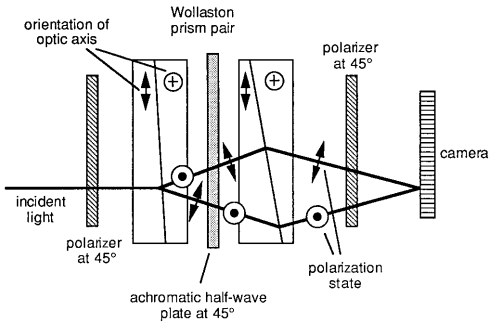

Fig. 5. Schematic layout of a wide-angle FT spectrometer based on two Wollaston prisms and an achromatic half-wave plate.

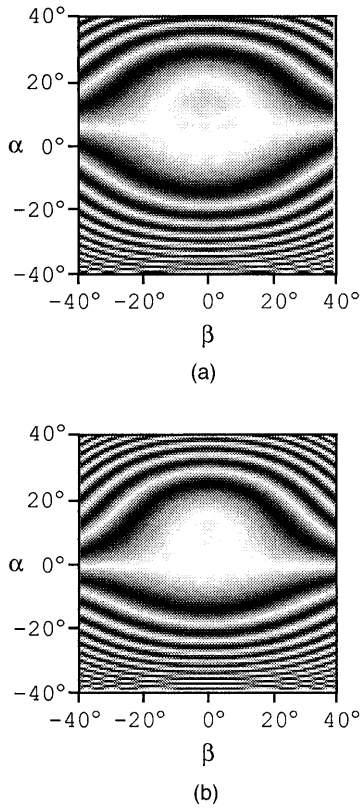

Fig. 6. Calculated field-of-view interferogram for different detector positions P in the fringe plane of the wide-angle spectrometer based on two Wollaston prisms and a half-wave plate: (a) P on axis corresponding to $\Delta_P(0°, 0°) = 0$, (b) P 3 mm off axis corresponding to $\Delta_P(0°, 0°) = 45$ μm.

the Wollaston prism or of material of the same sign, in which case a half-wave plate has to be inserted between the Wollaston prism and the compensation plate.

In the double-prism design, we used the second Wollaston prism not only to generate a fringe plane behind the prisms, but also to act as a compensation plate for the first prism. Figure 5 shows the layout of a wide-field-of-view FT spectrometer based on two Wollaston prisms that are made of crystals with the same sign of birefringence, separated by a half-wave plate. If one

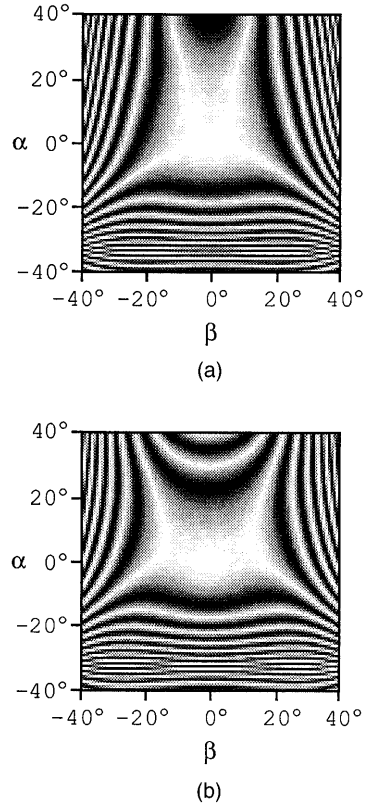

Fig. 7. Calculated field-of-view interferogram for different detector positions P in the fringe plane of the wide-angle spectrometer based on two Wollaston prisms fabricated from crystals with different signs of birefringence: (a) P on axis corresponding to $\Delta_P(0°, 0°) = 0$, (b) P 3 mm off axis corresponding to $\Delta_P(0°, 0°) = 45$ μm.

uses combinations of different birefringent materials of appropriate thicknesses, half-wave plates can be fabricated that are both achromatic and wide angle.[14]

Figure 6 shows field-of-view interferograms calculated for a spectrometer incorporating the same calcite Wollaston prisms as discussed in Section 4, combined with an achromatic, wide-angle half-wave plate. Figure 6(b) is calculated for an off-axis detector position and shows that the increased field of view is maintained over the full range of path differences. A comparison with Fig. 3 shows that the inclusion of a half-wave plate increases the solid-angle field of view by 2 orders of magnitude.

The range of input angles in our experimental arrangement (Fig. 2) is limited to ±15°, which prevents detailed confirmation of the predicted field-of-view interferogram. However, with an achromatic half-wave plate[15] and the Wollaston prism described above, we confirmed that the field of view of this combination exceeds ±15°.

Figure 7 shows field-of-view interferograms calculated for a combination of Wollaston prisms fabricated from calcite and rutile. To maximize the field of view in the visible region of the spectrum, Eq. (2) requires that the thickness of the rutile Wollaston prism has to be approximately 1.6 times that of the calcite prism. The first calcite Wollaston prism is

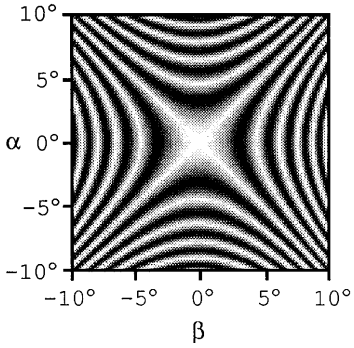

Fig. 8. Calculated field-of-view interferogram for the spectrometer based on a single Wollaston prism with inclined optic axis.

the same as used previously. The internal angle of the rutile Wollaston prism is set so as to maintain the path difference at ± 45 μm across the 6-mm aperture of the detector. A comparison with Fig. 4 again shows an increase in the solid-angle field of view by 2 orders of magnitude.

The inclusion of rutile into an optical system has two disadvantages: the cost and a short wavelength cutoff at 420 nm. A similar wide-angle combination for operation in the blue–UV region of the spectrum can be obtained by a combination of synthetic quartz and ammonium dihydrogen phosphate, which is useful for transmitting to 190 nm.

Finally, the fringe plane produced by a single Wollaston prism can be moved outside the prism if one inclines the optic axis in one of the component wedges[5]: As indicated by Eq. (1), the elimination of the second prism and reduction in overall thickness increases the field of view of the spectrometer.

Figure 8 shows the field-of-view interferogram calculated for an inclined-axis Wollaston prism. The internal angle within the Wollaston prism is 2.4°, and the optical axis in the first wedge is inclined by 12.4° with respect to the wedge interface. As above, this gives a path difference of ± 45 μm across the 6-mm aperture of the detector. Although the single Wollaston prism does not match the field of view of comparable pairs of Wollaston prisms that act as mutual compensation plates, the single prism does have a larger field of view than the conventional double Wollaston prism configuration and has the additional advantage of reducing the component count for any proposed instrument.

6. Conclusions

In this paper we presented new designs of static Fourier-transform spectrometers based on Wollaston prisms with significantly increased fields of view. Two of these configurations are predicted to possess fields of view larger than that of a comparable Fourier-transform spectrometer based on a Michelson interferometer.

By accurate numerical calculation of the field of view of various prism combinations, we have shown the benefits that can be obtained by the inclusion of an achromatic half-wave plate or by the combination of positive and negative birefringent materials for the fabrication of the Wollaston prisms. In addition, we modeled how a single Wollaston prism with an inclined optic axis produces a fringe plane localized behind its exit face, thus enabling the design of a static Fourier-transform spectrometer based on a single Wollaston prism.

The ability to calculate exactly the optimum position of the detector array allows a spectrometer to be designed in which the fringe plane is localized immediately behind the exit face of the prism. In principle, this enables a spectrometer to be designed with the Wollaston prisms, polarizers, and detector assembled as a solid block.[16]

Miles Padgett is a Royal Society Research Fellow.

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