# Single-pulse, Fourier-transform spectrometer having no moving parts 

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#### Abstract

A Wollaston prism is used in the design of a polarizing Fourier-transform spectrometer with no moving parts. The effective path difference between orthogonally polarized components varies across the aperture of the instrument, forming an interferogram in the spatial rather than temporal domain. The use of a charge-integrating linear detector array permits the entire interferogram to be sampled simultaneously so that a full spectrum is obtained for a single pulse of light. Key words: Fourier-transform spectroscopy, Wollaston prism, spatial interferogram, single-shot operation.


## Introduction

Conventional Fourier-transform spectrometers are based on Michelson interferometers. When the output of the interferometer is recorded as a function of the path diference between the two arms, an interferogram is obtained that is the autocorrelation of the optical field. The power spectrum of the Fourier transform of the interferogram corresponds to the spectral distribution of the input light. Two drawbacks associated with these instruments are that high-quality mirror-scanning mechanisms are required, and the temporal resolution is limited by the maximum mechanical scanning rate. Furthermore, the measurement of transient phenomena, such as short laser pulses and explosion spectra, requires the integration of data from many events. One can overcome these problems by using an interferometer in which the interferogram is formed in the spatial rather than the temporal domain. Researchers have demonstrated this by tilting the mirrors of a Michelson interferometer such that the wave fronts interfere at a small angle. The relative phase difference between phase fronts is then a function of the transverse displacement, so the autocorrelation function appears as a spatial intensity distribution. Stroke and Funkhouser ${ }^{1}$ recorded this pattern photographi-

[^0]cally and Fourier transformed it optically with coherent light to obtain the wavelength spectrum. Using current technology, one may use a photodiode array to detect the interferogram and a microcomputer to calculate its Fourier transform. ${ }^{2-4}$ The use of a Sagnac interferometer with common paths for both interferometer arms reduces the sensitivity to mechanical disturbances and air currents.

As an alternative, birefringent materials are frequently used as a way to introduce path differences between orthogonal polarization components of an incident light beam. One can subsequently make the two components interfere by passing the light through an analyzing polarizer. It is well known that when a Wollaston prism is placed between two suitably oriented polarizers and illuminated with a light source, a set of straight-line interference fringes will be produced localized to a plane within the prism. ${ }^{5}$ In Appendix A we show that these fringes are the Fourier transform of the spectral power distribution. Hence a Wollaston prism can be used as the basis of a Fourier-transform spectrometer.

## Principle of Operation

Here we report on a single-shot, Fourier-transform spectrometer that uses the birefringence of a Wollaston prism to introduce a path difference between orthogonal polarizations. The effective path difference is dependent on the lateral position within the Wollaston prism, and consequently an interferogram is obtained in the spatial domain. Because the path difference is introduced along a common path within a single optical element, our design is robust and insensitive to mechanical misalignment. Unlike previous designs, there are no beam splitters or mirrors;
with a suitable choice of components, no power is returned to the source. All optical components are held on a single optical axis, which leads to an extremely compact instrument. Our design has possible applications in a variety of fields, including environmental sensing, laser pulse diagnostics, coherent light detection, and spectral pattern recognition.
A Wollaston prism is formed from two birefringent wedges with their optic axes at right angles, both to each other and to the optical axis. Normally incident light is resolved in the first wedge into ordinary and extraordinary components that propagate collinearly with different phase velocities. At the interface between the wedges, the components are interchanged so that the ordinary component propagates as an extraordinary component and vice versa. In the center of the prism the effective path difference introduced between the orthogonal polarizations in the first wedge is canceled out by an equal and opposite effective path difference in the second wedge. Away from the central position the effective path difference, $\Delta$, is proportional to the displacement, $d$, and is given by ${ }^{5}$

$$
\begin{equation*}
\Delta=2 d\left(n_{e}-n_{o}\right) \tan \vartheta \tag{1}
\end{equation*}
$$

where $n_{o}$ and $n_{e}$ are the ordinary and extraordinary refractive indices and $\vartheta$ is the internal angle of the interface between the wedges. It is this effective path difference introduced by a Wollaston prism between orthogonal polarizations that forms the basis of the present Fourier-transform spectrometer design. The two polarization components undergo a symmetrical splitting at the wedge interface. The deviation angle, $\alpha$, between the two beams depends on $\vartheta$ and the birefringence of the prism material, and it is given by ${ }^{5}$

$$
\begin{equation*}
\alpha=2\left(n_{e}-n_{o}\right) \tan \vartheta . \tag{2}
\end{equation*}
$$

The design of the spectrometer is outlined in Fig. 1. The input light is linearly polarized at $45^{\circ}$ to the optic axes of the Wollaston prism, giving equal transmission intensities for the horizontally and vertically polarized components. A second $45^{\circ}$ polarizer placed after the Wollaston prism analyzes the transmitted light, permitting the two orthogonal polarizations to


Fig. 1. Fourier-transform spectrometer based on a Wollaston prism.
interfere. As we discussed above, the path difference between the two components depends on the lateral position across the Wollaston prism. The varying path difference across the prism results in the formation of interference fringes localized to a plane within the prism. A lens images this plane onto a linear photodiode array, and the resulting interferogram is recorded with a microcomputer. The output of each photodiode is proportional to the integral of the light intensity of a small element of the interferogram during the scan time of the photodiode array. This permits a complete interferogram to be simultaneously recorded for a single pulse of light.

The theoretical maximum resolution depends on the maximum path difference within the interferometer and on the form of apodization used. For triangular apodization, the FWHM instrumental linewidth, $\delta \sigma_{\text {FWHM }}$, is related to the maximum path difference, $\Delta L$, by ${ }^{6}$

$$
\begin{equation*}
\delta \sigma_{F W H M}=\frac{1.79}{2 \Delta L} . \tag{3}
\end{equation*}
$$

Application of the Nyquist criterion to the sampling interval of the interferogram implies a limit to the maximum valid spatial frequency, and hence we can deduce the shortest unambiguously measureable wavelength $\lambda_{\text {min }}$ to be

$$
\begin{equation*}
\lambda_{\min }=\frac{2 L_{\mathrm{tot}}}{N} \tag{4}
\end{equation*}
$$

where $L_{\text {tot }}$ is the effective path difference across the entire interferogram (for a symmetrical interferogram $L_{\text {tot }}=\Delta L$ ), and $N$ is the number of data points. This limit corresponds to the requirement that there be at least two data points per fringe within the interferogram.

## Experimental Arrangement

For a proof-of-principle demonstration of this technique, we used the experimental arrangement outlined below. The Wollaston prism is fabricated from calcite with a $20-\mathrm{mm}$ aperture and an internal angle, $\vartheta$, of $3^{\circ} .7$ We image the virtual fringe plane within the Wollaston onto a 1024 -element detector array by using two infinite-conjugate achromatic lenses (focal lengths 120 and 300 mm ). The magnification is 2.5 so that the central 10.24 mm of the Wollaston is imaged onto the $25.6-\mathrm{mm}$-wide detector array. The maximum path difference of approximately $100 \mu \mathrm{~m}$ gives a theoretical FWHM instrumental linewidth of $100 \mathrm{~cm}^{-1}$ for triangular apodization. For a 1024element detector array, the shortest unequivocal wavelength is 390 nm . The polarizers are of the simple Polaroid sheet type and are located in the pupil planes on either side of the Wollaston prism. For some applications it is desirable that no power is reflected back to the source. Under such conditions the polarizers can be angled slightly to the optical axis
so that reflected power is not returned to the source. The linear detector is a clocked photodiode array, with an update rate of 15 ms . The data are read into a personal computer, where the interferogram is recorded and the apodization and fast-Fouriertransform routines are applied to yield the spectral information.
The angular acceptance of the spectrometer is limited by the angular acceptance of the Wollaston prism. In addition to the path difference being a function of lateral position, the retardation between the two polarizations is also a function of angle, given by ${ }^{5}$

$$
\begin{equation*}
\delta \Delta=t \frac{n_{o}^{2}-n_{e}^{2}}{n_{o}^{2} n_{e}} \sin ^{2} i, \tag{5}
\end{equation*}
$$

where $\delta \Delta$ is the change in path difference with angle, $i$ is the angle of incidence, and $t$ is the thickness of the Wollaston wedges. For our Wollaston arrangement this limits the angular acceptance to a few degrees. However, we can increase this angle by using a wide-field Wollaston design comprising two identical Wollastons separated by an achromatic half-wave plate. ${ }^{5}$

The spatial frequency of the fringes, $v_{d}$, in the plane of the camera is related to the birefringence, $n_{e}-n_{o}$, the magnification of the imaging lenses, $M$, and the wavelength, $\lambda$, of the light by

$$
\begin{equation*}
v_{d}=\frac{2\left(n_{e}-n_{o}\right) \tan \vartheta}{\lambda M} . \tag{6}
\end{equation*}
$$

The birefringence and the magnification are both functions of wavelength. The birefringence of the calcite across the wavelength range of the instrument varies from 0.186 at 390 nm to 0.164 at $1 \mu \mathrm{~m} .{ }^{8}$ The magnification of the relay lenses was measured to vary by $1.6 \%$ over the same wavelength range. Both of these effects slightly distort the wavelength scale in the transformed spectra, but they can be accurately corrected by the incorporation of a factor that is the


Fig. 2. Fast fourier transform of a $632.8-\mathrm{nm}$ helium-neon interferogram.
ratio of $\left(n_{e}-n_{o}\right) / M$ at the measured wavelength compared with the calibration wavelength. We have confirmed the wavelength calibration of the instrument to within 0.2 nm by using helium-neon lasers operating at 632.8 and 543.5 nm . In both cases the instrumental resolution was measured to be $\approx 100$ $\mathrm{cm}^{-1}(4 \mathrm{~nm}$ at 632.8 nm$)$, which agrees with the theoretical predictions. An example of the Fouriertransformed spectrum for a helium-neon laser at 632.8 nm is reproduced in Fig. 2.

Ideally, the two components interfering at the detector emanate from the same point within the Wollaston prism, in which case the degree of spatial coherence of the light at the Wollaston is not important. However, if there is a residual focus error, the two components effectively emanate from slightly different positions at the plane of the Wollaston. Once the separation exceeds the distance over which the light is spatially coherent, there will be a significant loss of fringe contrast in the transform plane. This effect could be usefully employed as a method for


Fig. 3. Interferogram obtained from a metal halide white-light source.


Fig. 4. Fourier transform of Fig. 3, showing the spectral properties of the light source.
detecting spatial coherence, but for a broadband spectrometer it places a limit on the chromatic aberration of the relay lenses that can be tolerated. The most demanding test is then to form simultaneous broadband interference fringes with an extended white-light source. Figure 3 shows such an interferogram for an extended metal halide lamp source in which achromatic relay lenses have been used. The prominent fringes near zero path difference result from the broadband nature of the light source, and the fringes at large path differences are due to the discrete emission lines within the spectrum.

The Fourier transform of the interferogram is shown in Fig. 4 and demonstrates that fringes are formed simultaneously for wavelengths between 400 and 1000 nm . The presence of emission lines illustrates the spectroscopic applications of this instrument. In particular, the xenon line at 823 nm and the unresolved sodium-D lines at 590 nm can be identified clearly. The spectral coverage is limited by the transmission characteristics of the optical components and the detectivity of the silicon detector array. In principle, we could extend it by using wider bandwidth polarizers and a cement-free Wollaston prism (e.g., magnesium fluoride is birefringent and transmits from 150 nm to $7.5 \mu \mathrm{~m}$ ). The ultimate bandwidth of the instrument is limited only by the range over which multielement detector arrays can be obtained.

In addition, we have used the flash lamp as the light source and a series of calibrated bandpass interference filters as the test objects ( 440,560 , and 680 nm ). The flash lamp and the detector are synchronized so that a complete interferogram is recorded for a single flash of the lamp. Fig. 5 shows the interferogram obtained with the $560-\mathrm{nm}$ filter; its Fourier transform is reproduced in Figure 6. As with the other filters, the center wavelength and the optical bandwidth agree with the calibration measurements to within less than 1 nm .


Fig. 5. Interferogram for a 10 -nm-wide bandpass interference filter centered at 560 nm .


Fig. 6. Fast Fourier transform of Fig. 5.
The optical signal-to-noise ratio of our system is limited to approximately 30 dB by the dynamic range of the detector array, although the repeatability of the technique permits this to be increased by signal averaging. After calibration of fixed pattern noise, the ultimate signal-to-noise ratio is determined by the cleanliness of the optics, particularly the Wollaston prism. Low-intensity artifacts can appear because of detector array clocking noise, and harmonic distortion products introduced by saturation of the photodiodes can be aliased into the spectrum.

One feature of this spectrometer design is that the spatial intensity of the interferogram is multiplied by the illumination profile at the plane of the Wollaston prism. In the spectral domain, this results in the effective instrumental resolution being the convolution of the Fourier transform of the illumination profile with the idealized instrumental resolution function. To obtain accurate spectra we therefore desire to have a spectrally uniform intensity profile at the Wollaston.

## Conclusions

We have demonstrated a single-shot, Fourier-transform spectrometer that has no moving parts. The calibration of the instrument is geometrically determined and therefore does not require a reference wavelength. A common optical path for both paths within the interferometer offers the advantage that the instrument is insensitive to mechanical vibration and air currents. Its resolution is a function of the maximum path difference, and one could increase this by using a larger internal angle for the calcite Wollaston. The scale of the interferogram is determined by the optical magnification of the system, and therefore a more sophisticated lens arrangement would permit the resolution and wavelength range to be varied within a single instrument.

## Appendix A. Derivation of Interferogram

Consider light propagating in the $z$ direction incident normally upon a Wollaston prism. The light is lin-
early polarized with its plane of polarization at an angle of $45^{\circ}$ to the axis of the prism, which is taken to lie in the $x$ direction. The time variation of the incident optical field can be expressed in terms of its $x$ and $y$ components as

$$
\begin{equation*}
\mathbf{E}(t)=\hat{i} \int_{-\infty}^{\infty} A(\omega) \exp (i \omega t) \mathrm{d} \omega+\hat{j} \int_{-\infty}^{\infty} A(\omega) \exp (i \omega t) \mathrm{d} \omega, \tag{7}
\end{equation*}
$$

where $\hat{i}$ and $\hat{j}$ are unit vectors in the $x$ and $y$ directions, respectively, and $A(\omega)$ is the Fourier amplitude corresponding to angular frequency $\omega$.
At distance $d$ from the central position of the Wollaston prism, the two halves of the Wollaston prism differ in thickness by $l$. The difference between the optical path lengths $\Delta$ traversed by the $x$ - and $y$-polarized components is simply written as

$$
\begin{equation*}
\Delta=p_{y}-p_{x}=\left(n_{e}-n_{o}\right) l, \tag{8}
\end{equation*}
$$

where $l$ depends on the internal angle of the Wollaston, and the displacement from the central position as before $\Delta$ can be written as

$$
\begin{equation*}
\Delta=2 d\left(n_{e}-n_{o}\right) \tan \vartheta \tag{9}
\end{equation*}
$$

After passing through the Wollaston prism, the $x$ and $y$ components follow slightly different ray paths before recombining at the detector array. The linear polarizer placed after the prism has its axis at an angle of $45^{\circ}$ to the $x$ axis. It transmits the component of the $x$ and $y$ polarizations at $45^{\circ}$ to the $x$ axis. Taking into account the effect of this linear polarizer and the phase delay introduced by the optical path length between the prism and the detector, we find that the amplitude of the optical field at the detector array is

$$
\begin{align*}
E(t)= & \frac{1}{\sqrt{2}} \int_{-\infty}^{\infty} A(\omega) \exp \left[i \omega\left(t-p_{x} / c\right)\right] \mathrm{d} \omega \\
& +\frac{1}{\sqrt{2}} \int_{-\infty}^{\infty} A(\omega) \exp \left[i \omega\left(t-p_{y} / c\right)\right] \mathrm{d} \omega . \tag{10}
\end{align*}
$$

The signal $S$ recorded by the detector array is proportional to the intensity integrated over the time of observation so that

$$
\begin{equation*}
S=\frac{k}{4 \pi} \int_{-\infty}^{\infty} E(t) E^{*}(t) \mathrm{d} t \tag{11}
\end{equation*}
$$

where * denotes the complex conjugate, $k$ is a constant, and we have extended the limits of the integra-
tion to $\pm \infty$ by defining $E(t)=0$ for times outside the time of observation.

Substituting Eq. (10) into Eq. (11) gives

$$
\begin{align*}
S= & \frac{k}{8 \pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(\omega) A^{*}(\omega)\left\{\exp \left[i \omega\left(t-p_{x} / c\right)\right]\right. \\
& \times \exp \left[-i \omega^{\prime}\left(t-p_{x} / c\right)\right] \\
& +\exp \left[i \omega\left(t-p_{y} / c\right)\right] \exp \left[-i \omega^{\prime}\left(t-p_{y} / c\right)\right] \\
& +\exp \left[i \omega\left(t-p_{x} / c\right)\right] \exp \left[-i \omega^{\prime}\left(t-p_{y} / c\right)\right] \\
& \left.+\exp \left[i \omega\left(t-p_{y} / c\right)\right] \exp \left[-i \omega^{\prime}\left(t-p_{x} / c\right)\right]\right\} \mathrm{d} \omega \mathrm{~d} \omega^{\prime} \mathrm{d} t . \tag{12}
\end{align*}
$$

Carrying out the time integration, making use of the Dirac delta function, $\left(=1 / 2 \pi \int_{-\infty}^{\infty} \exp (i \omega t) \mathrm{d} t\right)$, and simplifying yield

$$
\begin{align*}
S & =\frac{k}{2} \int_{-\infty}^{\infty}|A(\omega)|^{2}\left\{1+\cos \left[\frac{\omega\left(p_{y}-p_{x}\right)}{c}\right]\right\} \mathrm{d} \omega \\
& =k \int_{0}^{\infty}|A(\omega)|^{2}\left\{1+\cos \left[\frac{\omega\left(p_{y}-p_{x}\right.}{c}\right]\right\} \mathrm{d} \omega \tag{13}
\end{align*}
$$

is which the last step makes use of the fact that the integrand on the right is an even function of $\omega$.
Writing $S_{0}=k_{0} \int_{0}^{\infty}|A(\omega)|^{2} \mathrm{~d} \omega$, where $S_{0}$ represents the signal for large optical path difference, and using Eq. (9), we see that it follows from Eq. (13) that

$$
\begin{equation*}
\frac{S-S_{0}}{S_{0}}=\frac{\int_{0}^{\infty}|A(\omega)|^{2} \cos \left[\omega \frac{2 d\left(n_{e}-n_{o}\right) \tan \vartheta}{c}\right] \mathrm{d} \omega}{\int_{0}^{\infty}|A(\omega)|^{2} \mathrm{~d} \omega} \tag{14}
\end{equation*}
$$

In the ideal case, when the birefringence ( $n_{e}-n_{o}$ ) does not depend on frequency $\omega$, the right-hand side of Eq. (14) is exactly the normalized Fourier cosine transform of the power spectrum $|A(\omega)|^{2}$ of the incident optical field, with the quantity $2 d\left(n_{e}-n_{o}\right)$ $\tan \vartheta / c$ acting as the time variable. As we showed experimentally, when Fourier transforming the interferogram that results from the variation of $S$ with $d$ to find $|A(\omega)|^{2}$, we find that the frequency dependence of ( $n_{e}-n_{o}$ ) results only in a slight distortion of the frequency scale, which can be compensated for by suitable calibration.

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