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Field Homogenizing Coils for Nuclear Spin Resonance Instrumentation

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The coils described in this paper are assembled in two relatively thin flat stacks which are placed on each pole piece of a nuclear spin resonance magnet. One coil of each coil pair is in each of the two stacks, and each coil pair is so designed that the current circulating in it produces, in the airgap center, a correcting magnetic field describable by one of a set of spherical harmonics. The orthogonal properties of the spherical harmonics render possible substantially independent adjustments of the currents in the several coil pairs, when some spin resonance parameter such as line width is utilized to determine that individual optima are reached.

Experimental coil stacks with 13 coil pairs, for the second, third, and zonal fourth harmonics, have been constructed, and preliminary results obtained by Rex Richards (Oxford) indicate that when a 4-mm spin sample is used, this method can serve to reduce field inhomogeneities to less than 1 part in 10^8 with few sets of 13 current adjustments.

THE work summarily reported here originated with the surmise that if a set of accessory coils could serve to produce, about a point within a magnetic air gap, incremental magnetic fields derivable from the various orthogonal spherical harmonics of several ascending orders, then the currents in these coils could be adjusted independently so as to maximize the nuclear spin resonance resolving power obtainable for a spherical sample centered at that point. It was also surmised that the several current adjustments would be nearly independent of each other for massive but nonspherical samples.

The general arrangement suggested above can be thought of as an extension of the degaussing coils used in ships. The main field is not affected, but the field inhomogeneities are "degaussed" by coils which operate orthogonally to each other.

A first set of coils was wound on a ping-pong ball, with provisions for inserting a sample in its center. This set consisted of 13 coils, 5 for the second, and 7 for the third orders, and the 13th coil for the zonal harmonic of the fourth order. The usual z axis of the spherical harmonics was the magnet axis.

This first set was never tried for its originally intended purpose, because the presence of a general obstruction within the gap was awkward and unsuitable for cryostatic operation. Nevertheless, this first set of coils was found useful to check the orthogonality of the later sets of coils.

A second set, consisting of 13 pairs of coils, was embroidered with copper wire on silk crepe stretched on two circular embroidery hoops. The two coils of each pair were series connected, with one coil on each crepe carrier, each pair being designed to produce the incremental magnetic field derivable from one of the 13 harmonics enumerated above.

The two subsets of 13 coils each forming the complete set were imbedded within a plastic and formed two paddle-like assemblies which could be conveniently placed against the pole pieces of a permanent magnet, leaving an unobstructed working space in the air-gap center. This second set was tried in collaboration with J. G. Atwood of The Perkin-Elmer Corporation and N. I. Adams of Nuclear Magnetic Corporation, and gave sufficiently promising results to warrant further work.

In the third set the wire-in-crepe arrangement was replaced by coils made as photoetched circuits, and stacked to form two assemblies, each 0.2 in. thick, one for each pole face.

The calculating method to determine the correct geometry of the current paths in the coils was as follows: Let r, φ, z, and z_0 represent the distance from the magnet axis, the azimuth about this axis, the cartesian ordinate along this axis, and the half-width of the air gap, respectively. The Laplacian of the general expression

$$J_m\left(\alpha_i\frac{r}{z_0}\right)\begin{Bmatrix}\sinh\left(\alpha_i\frac{z}{z_0}\right)\\ \cosh\left(\alpha_i\frac{z}{z_0}\right)\end{Bmatrix}\begin{Bmatrix}\cos m\varphi\\ \sin m\varphi\end{Bmatrix} \quad (1)$$

vanishes identically, and the mathematical circumstance that this expression becomes quiescent at and near $z=0$ when r approaches infinity makes it a suitable basis for a magnetic potential function requiring no boundary conditions (coils with currents) in the median plane of the air gap.

When $m=0$, for example, the expression (1) with cosh contains all zonal harmonics of even orders. Let there be three values of i. If, in the expression

$$\Phi_{2,0}=\sum_{i=1}^{i=3} a_i J_0\left(\alpha_i\frac{r}{z_0}\right)\cosh\left(\alpha_i\frac{z}{z_0}\right), \quad (2)$$

the a_i's are the following functions of the α_i's,

$$a_1=\frac{1}{\alpha_1^4}(\alpha_2^2-\alpha_3^2),\quad a_2=\frac{1}{\alpha_2^4}(\alpha_3^2-\alpha_1^2),\quad a_3=\frac{1}{\alpha_3^4}(\alpha_1^2-\alpha_2^2), \quad (3)$$

the expression thus obtained constitutes a suitable basis for the design of coils producing in the air-gap center the field derivable from the zonal harmonic of second order and designed to cancel the so-called coning distortion of the magnetic field, since it contains the second-order harmonic,

$$\left[\frac{1}{\alpha_1^2}(\alpha_2^2-\alpha_3^2)+\frac{1}{\alpha_2^2}(\alpha_3^2-\alpha_1^2)+\frac{1}{\alpha_3^2}(\alpha_1^2-\alpha_2^2)\right]\frac{2z^2-r^2}{4z_0^2}, \quad (4)$$

and, furthermore, contains no fourth- and no sixth-order harmonic, while the terms of the next higher order harmonic are eighth-order terms with very small coefficients.

The final selection of the α_i's for this case ($n=2$, $m=0$), and similarly for all the other cases thus treated, was determined by the stipulation that the power dissipation within the coils be an approximate minimum for a given field correction. This will be explained below for this particular case, but a word should be said first about the determination of the coil configurations from the potential function defined by (2) and (3).

At the pole faces, there will be no discontinuity in the magnetic induction normal to these faces, and the boundary conditions will be satisfied by making the current density within the flat coil on each pole face proportional to and perpendicular to the projection of the magnetic vector on that pole face. Since this projection is the gradient of the potential function in the plane of the pole face, if a sheet of conducting material, such as one copper sheet of a photoetching laminate, is cut along the contours of this potential function, and if currents of equal magnitude are caused to circulate in each conducting band between two adjacent contour lines, this will generate the magnetic field derivable from that particular potential function.

At $z=z_0$, the potential function approximating in the air-gap center the zonal harmonic of second order which is treated here has the value

$$\Phi_{2,0}(r)=\frac{1}{\alpha_1^4}(\alpha_2^2-\alpha_3^2)J_0\left(\alpha_1\frac{r}{z_0}\right)\cosh\alpha_1+\cdots. \quad (5)$$

The power dissipated by the electrical currents producing this harmonic is proportional to

$$\int\left(\frac{\partial\Phi_{2,0}(r)}{\partial r}\right)^2 rdr. \quad (6)$$

The integrand contains the square terms $\{(\partial/\partial r)J_0[\alpha_i(r/z_0)]\}^2$, etc., and cross products. In order to simplify the calculations, the integrands involving cross products and having positive and negative values were considered negligible by comparison with the integrands of the form $\int\{(\partial/\partial r)J_0[\alpha_i(r/z_0)]\}^2 rdr$. Furthermore, all three of the latter were assumed to have equal values, which was equivalent to arresting their integration at some common homologous point of the Bessel function. These admittedly drastic simplifications were thought permissible because rigorous power dissipation minima were not essential to the problem, and because the dominant character of the high powers of the α's and of the hyperbolic functions in (7) and (10), and in the other similar expressions for the other cases, serves to minimize the effect of these assumptions. The α_i's were then determined by minimizing the ratio

$$\frac{\frac{1}{\alpha_1^8}(\alpha_2^2-\alpha_3^2)^2\cosh^2\alpha_1+\frac{1}{\alpha_2^8}(\alpha_3^2-\alpha_1^2)^2\cosh^2\alpha_2+\frac{1}{\alpha_3^8}(\alpha_1^2-\alpha_2^2)^2\cosh^2\alpha_3}{\left[\frac{1}{\alpha_1^2}(\alpha_2^2-\alpha_3^2)+\frac{1}{\alpha_2^2}(\alpha_3^2-\alpha_1^2)+\frac{1}{\alpha_3^2}(\alpha_1^2-\alpha_2^2)\right]^2} \quad (7)$$

in which the numerator is the radically simplified expression which is proportional to the electrical energy dissipated in the coil, and the denominator is the square of the expression (4) which is proportional to the intensity of the magnetic field generated.

The ratio has six minima, corresponding to the six permutations of the three α_i's which have the values, 1.46, 4.15, and 8.16. These values were inserted in (5) for the determination of the coils for the $n=2$, $m=0$ harmonic.

The value of the steps for the determination of the actual contour line density was chosen as small as possible without bringing the successive photoetched coil turns closer than 0.040 in., in order to obtain the maximum coil impedance realizable with a single photoetched copper layer.

As another example, if in the expression

$$\Phi_{3,1}=\sum_{i=1}^{i=3} a_iJ_1\left(\alpha_i\frac{r}{z_0}\right)\cosh\left(\alpha_i\frac{z}{z_0}\right)\begin{Bmatrix}\cos\varphi\\ \sin\varphi\end{Bmatrix}, \quad (8)$$

the a_i's are given by the following functions of the α_i's,

$$a_1=\frac{1}{\alpha_1}(\alpha_2^4-\alpha_3^4),\quad a_2=\frac{1}{\alpha_2}(\alpha_3^4-\alpha_1^4),\quad a_3=\frac{1}{\alpha_3}(\alpha_1^4-\alpha_2^4), \quad (9)$$

the first (cross-field) and fifth-order terms in $\cos\varphi$ and $\sin\varphi$ will have zero coefficients, and the α_i's may then be optimized for least power dissipation by minimizing the expression

$$\frac{\frac{1}{\alpha_1^2}(\alpha_2^4-\alpha_3^4)^2\cosh^2\alpha_1+\frac{1}{\alpha_2^2}(\alpha_3^4-\alpha_1^4)^2\cosh^2\alpha_2+\frac{1}{\alpha_3^2}(\alpha_1^4-\alpha_2^4)^2\cosh^2\alpha_3}{[\alpha_1^2(\alpha_2^4-\alpha_3^4)+\alpha_2^2(\alpha_3^4-\alpha_1^4)+\alpha_3^2(\alpha_1^4-\alpha_2^4)]^2}. \tag{10}$$

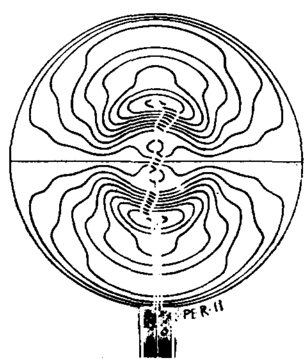
The function thus obtained may be used to approximate the field derivable from the tesseral harmonic of third order and first kind ($n=3$, $m=1$).

After the α_i's have been determined, for all $\Phi_{n,m}$'s decided upon, the a_i's determined from the α_i's are given a final small correction which consists in dividing them by $\cosh(\alpha_i t/z_0)$, where t designates the small distance from the pole face where the coil being calculated will be located.

Altogether there will be 8 functions similar to (2) and (8) with coefficients corrected as previously indicated; 3 for the second order, 4 for the third-order, and the 8th-function for the single 4th-order zonal harmonic.

Figure 1 illustrates the etched blank from which are cut the flat coils for the tesseral harmonics defined by $n=2$, $m=1$. The current access and return are provided by simple paths etched on the reverse side of the copper clad plastic sheet, and joining the two "sinks" with the connection tabs.

FIG. 1. Coil for tesseral harmonic of the second order and first kind.



As already indicated, the coil pair for each harmonic is made up of two identical series connected coils, such as the $n=2$, $m=1$ coil illustrated in Fig. 1, for instance, one of any pair to each stack. A single coil pair is provided for each zonal harmonic, but two coil pairs are provided for all others. Thus, there is a second $n=2$, $m=1$ coil pair, at 90° to the first pair. The same remark applies to the two $n=3$, $m=1$ coil pairs. The two $n=2$, $m=2$ harmonics are generated by two coil pairs which are at 45° to each other, and the same remark applies to the $n=3$, $m=2$ harmonics. Finally, the $n=3$, $m=3$ sectional harmonics are generated by two coil pairs which are at 30° to each other.

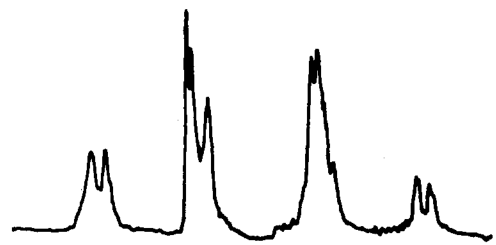


FIG. 2. Quadruplet of the nuclear spin resonance spectrum of alcohol

The direction of current in any two paired coils is the same when the sinh function is used for the potential function, i.e., whenever $n+m$ is odd, and is different when the cosh function is used, i.e., whenever $n+m$ is even.

The writer is indebted to Rex Richards, of Lincoln College, Oxford, for the record of the quadruplet of the ethanol spectrum shown in Fig. 2. This record was obtained with a 4-mm diameter cylindrical sample spun in a $1\frac{1}{2}\times 8$ in. permanent magnet air gap equipped with the set of 13 homogenizing coils designed as described above. The two resonance peaks shown resolved in the second band are 0.15 cps apart.

Richards reports that the current adjustments required to obtain this result can be effected with relative dispatch, because the individual adjustments have been found to be substantially noninteracting, as had been surmised.